Lecture 3

2018/2019

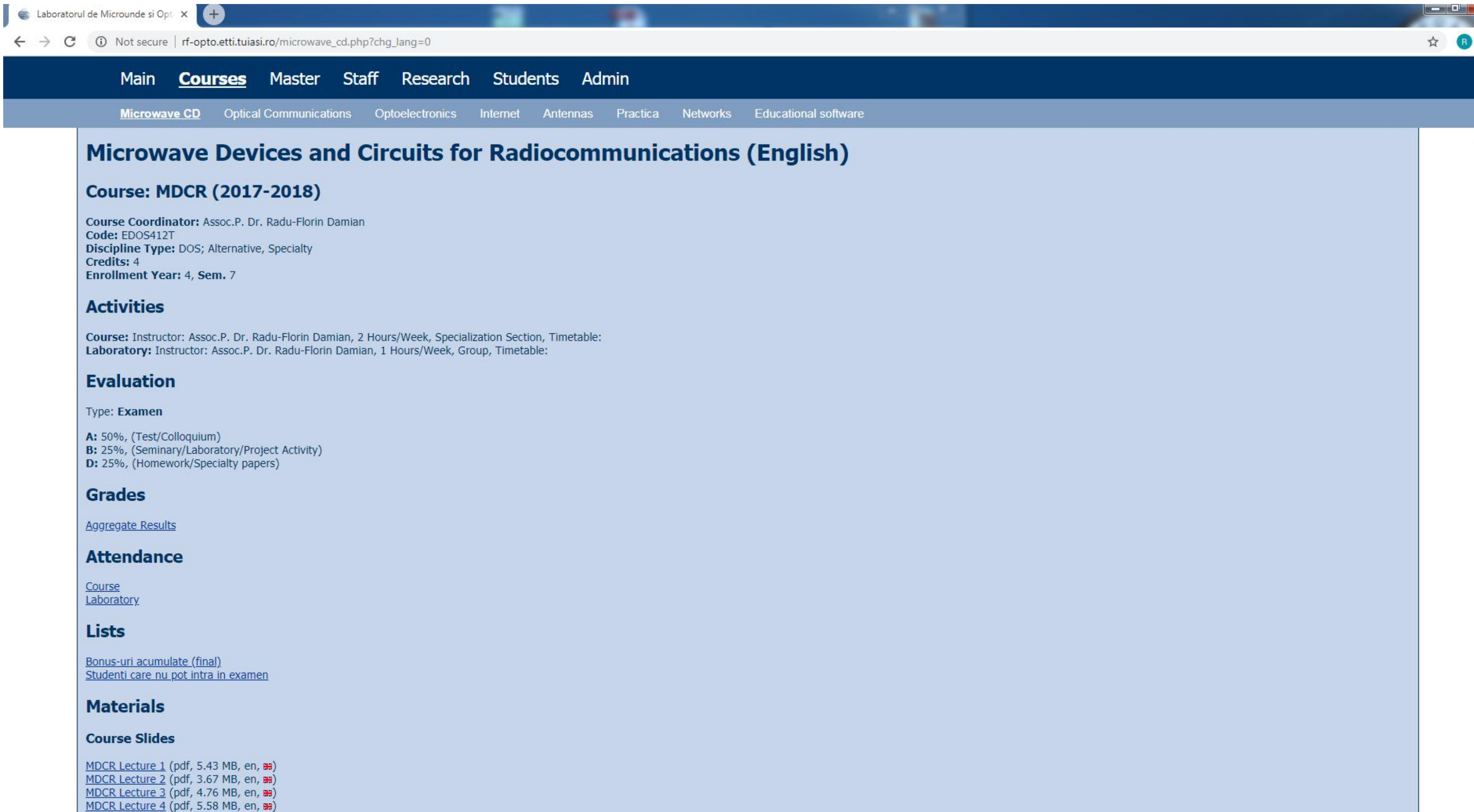
Microwave Devices and Circuits for Radiocommunications

2018/2019

- 2C/1L, **MDCR**
- **Attendance at minimum 7 sessions (course + laboratory)**
- Lectures- **associate professor Radu Damian**
  - Friday 09-11, ? III.34, II.13
  - E – 50% final grade
  - **problems** + (2p atten. lect.) + (3 tests) + (bonus activity)
    - 3p=+0.5p
  - **all materials/equipments authorized**
- Laboratory – **associate professor Radu Damian**
  - Wednesday 12-14, II.12 odd weeks
  - L – 25% final grade
  - P – 25% final grade

Materials

- http://rf-opto.etti.tuiasi.ro



Laboratorul de Microunde si Opt

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Main Courses Master Staff Research Students Admin

Microwave CD Optical Communications Optoelectronics Internet Antennas Practica Networks Educational software

Microwave Devices and Circuits for Radiocommunications (English)

Course: MDCR (2017-2018)

Course Coordinator: Assoc.P. Dr. Radu-Florin Damian
Code: EDOS412T
Discipline Type: DOS; Alternative, Specialty
Credits: 4
Enrollment Year: 4, **Sem.** 7

Activities

Course: Instructor: Assoc.P. Dr. Radu-Florin Damian, 2 Hours/Week, Specialization Section, Timetable:
Laboratory: Instructor: Assoc.P. Dr. Radu-Florin Damian, 1 Hours/Week, Group, Timetable:

Evaluation

Type: **Examen**

A: 50%, (Test/Colloquium)
B: 25%, (Seminary/Laboratory/Project Activity)
D: 25%, (Homework/Specialty papers)

Grades

Aggregate Results

Attendance

Course
Laboratory

Lists

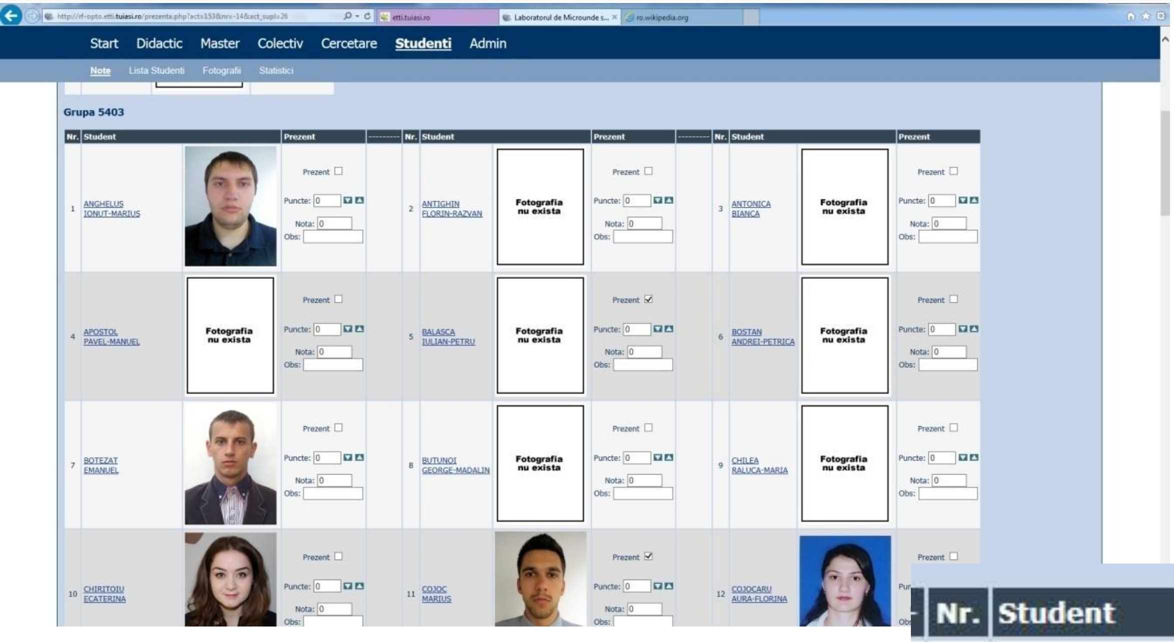
Bonus-uri acumulate (final)
Studenti care nu pot intra in examen

Materials

Course Slides

MDCR Lecture 1 (pdf, 5.43 MB, en,)
MDCR Lecture 2 (pdf, 3.67 MB, en,)
MDCR Lecture 3 (pdf, 4.76 MB, en,)
MDCR Lecture 4 (pdf, 5.58 MB, en,)

Photos



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Access

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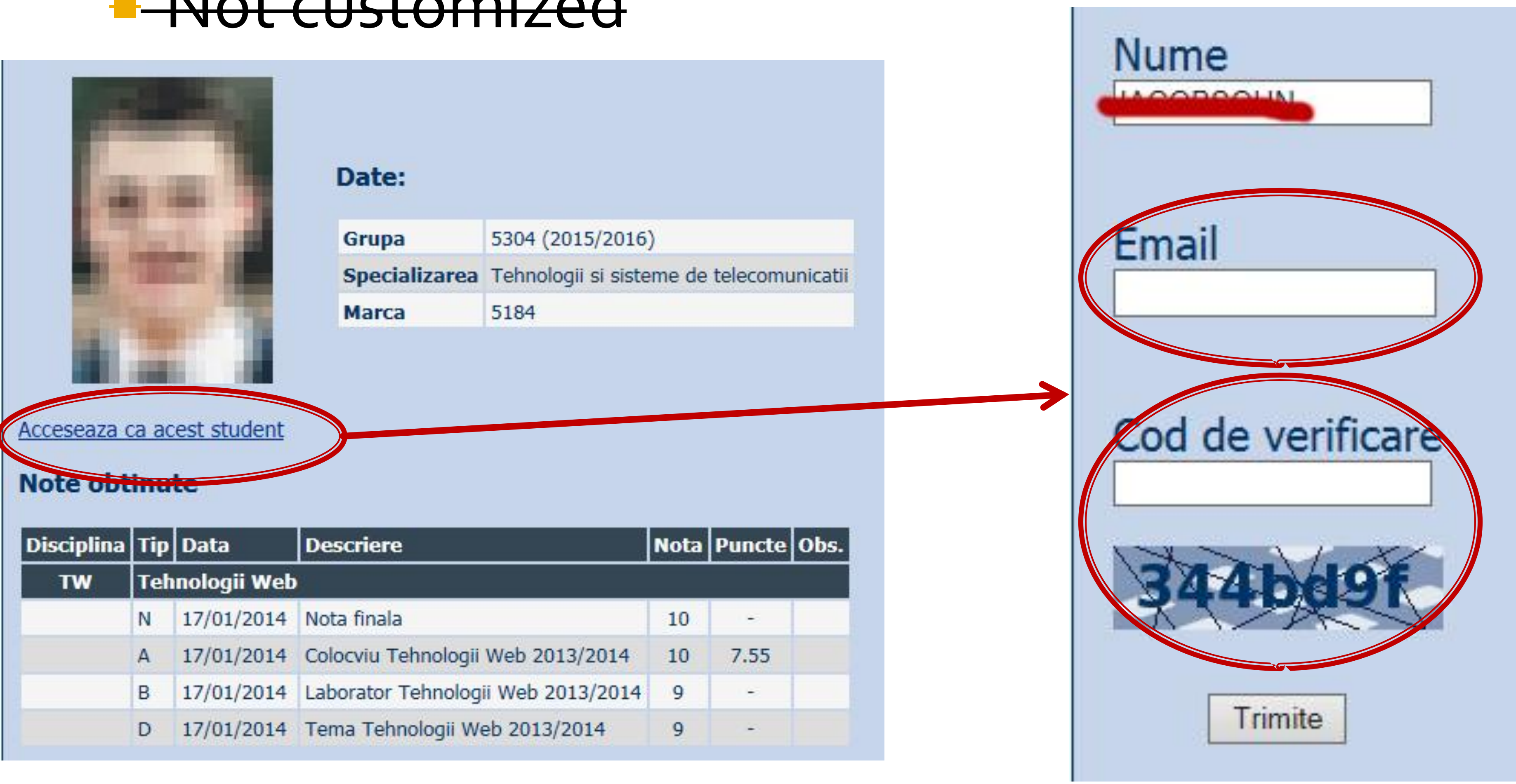
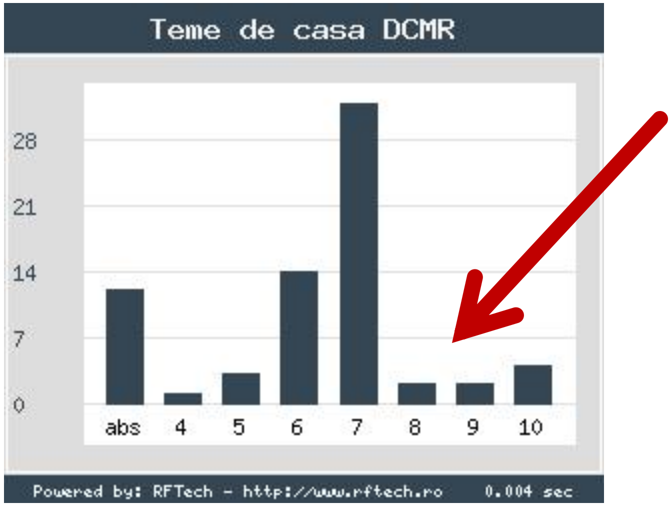


Proiect 2018/2019

- factorul "andrei" = -2p

2017/8



Examen: Logarithmic scales

dB $= 10 \cdot \log_{10} (P_2 / P_1)$

0 dB	= 1
+ 0.1 dB	= 1.023 (+2.3%)
+ 3 dB	= 2
+ 5 dB	= 3
+ 10 dB	= 10
-3 dB	= 0.5
-10 dB	= 0.1
-20 dB	= 0.01
-30 dB	= 0.001

dBm $= 10 \cdot \log_{10} (P / 1\text{ mW})$

0 dBm	= 1 mW
3 dBm	= 2 mW
5 dBm	= 3 mW
10 dBm	= 10 mW
20 dBm	= 100 mW
-3 dBm	= 0.5 mW
-10 dBm	= 100 μW
-30 dBm	= 1 μW
-60 dBm	= 1 nW

[dBm] + [dB] = [dBm]

[dBm/Hz] + [dB] = [dBm/Hz]

[x] + [dB] = [x]

Examen

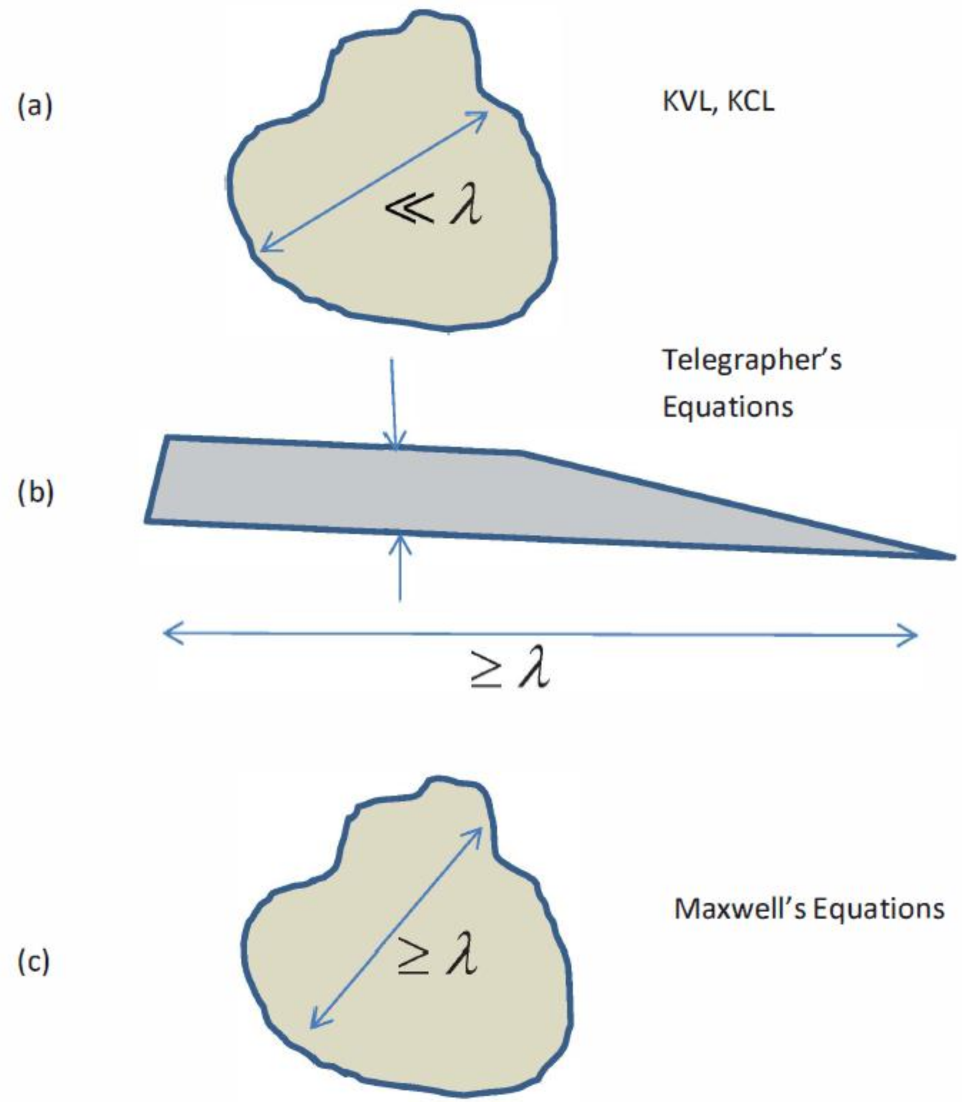
- Complex numbers arithmetic**!!!!**
- $z = a + j \cdot b$; $\mathbf{j^2 = -1}$

Introduction

Electrical Length

- Behavior (and description) of any circuit depends on his electrical length at the particular frequency of interest
  - E≈o → Kirchhoff
  - E>o → wave propagation

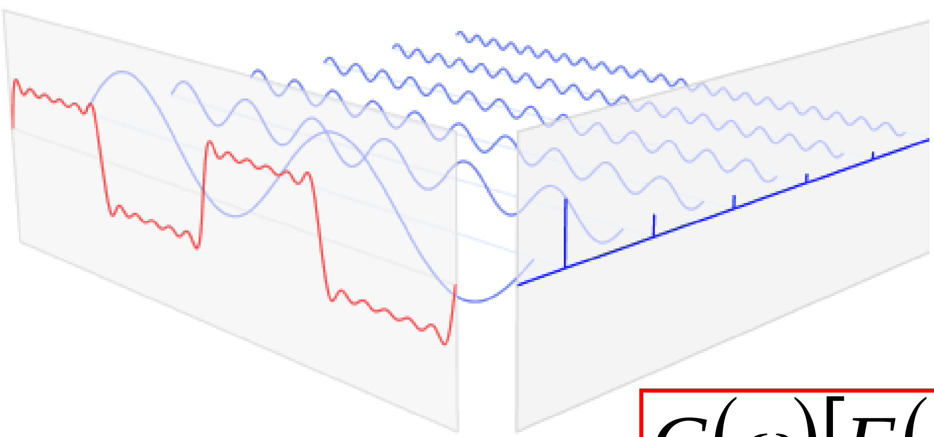
$$E = \beta \cdot l = \frac{2\pi}{\lambda} \cdot l = 2\pi \cdot \left(\frac{l}{\lambda}\right)$$



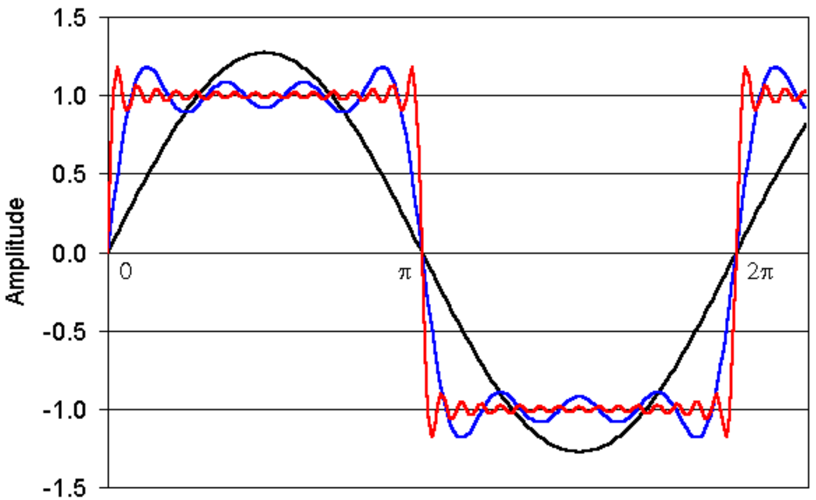
Mathematical models

- particular cases where analytical solution exists
  - harmonic signals, Fourier Transform, frequency spectrum

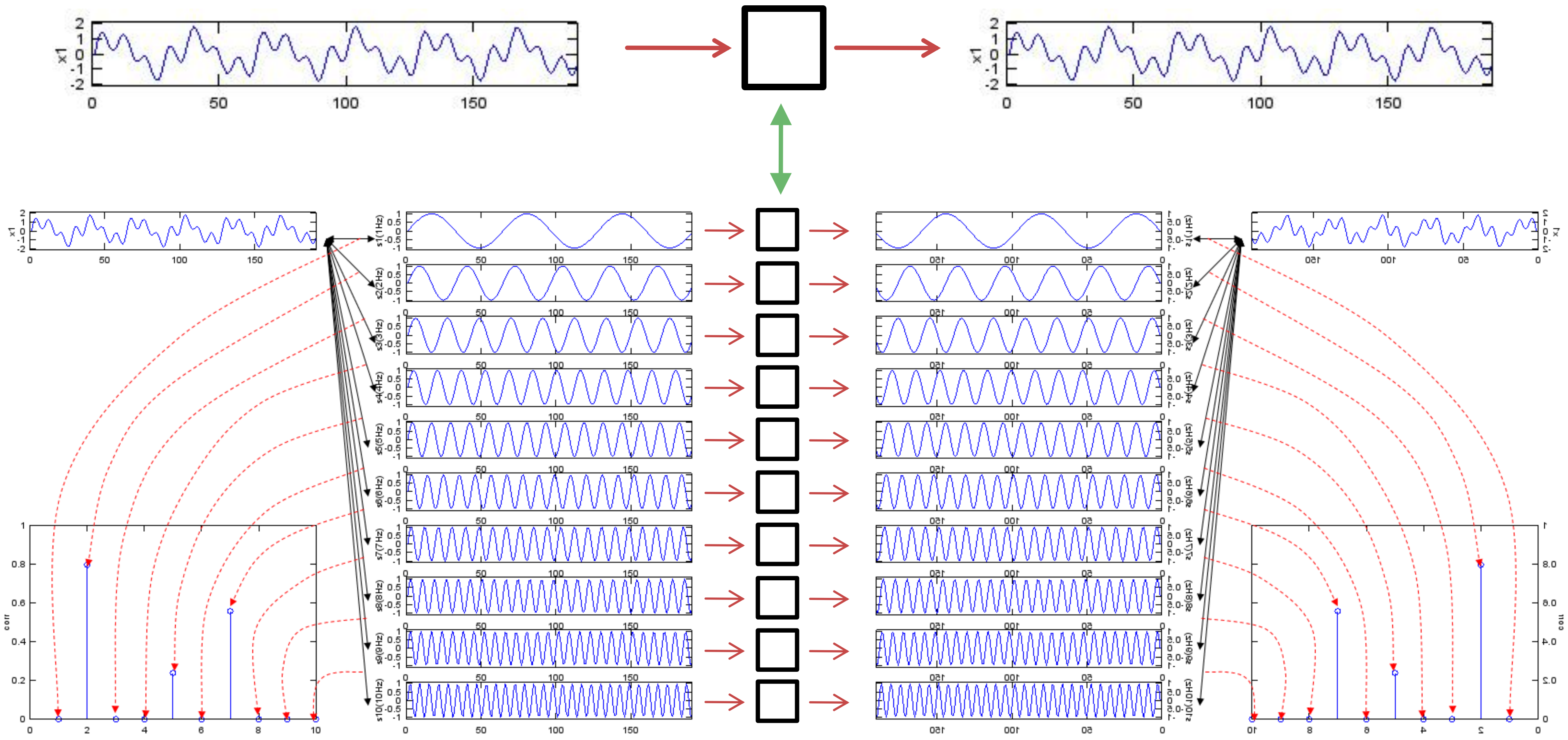
$$X = X_0 e^{j\cdot\omega\cdot t} \qquad \frac{\partial X}{\partial t} = j\cdot\omega\cdot X \qquad g(\omega) = \int_{-\infty}^{\infty} f(t)\cdot e^{-j\omega t}\,dt \qquad f(t) = \int_{-\infty}^{\infty} g(\omega)\cdot e^{j\omega t}\,d\omega$$



$$\boxed{G(\omega)[F(\omega)]}$$



Mathematical modeling

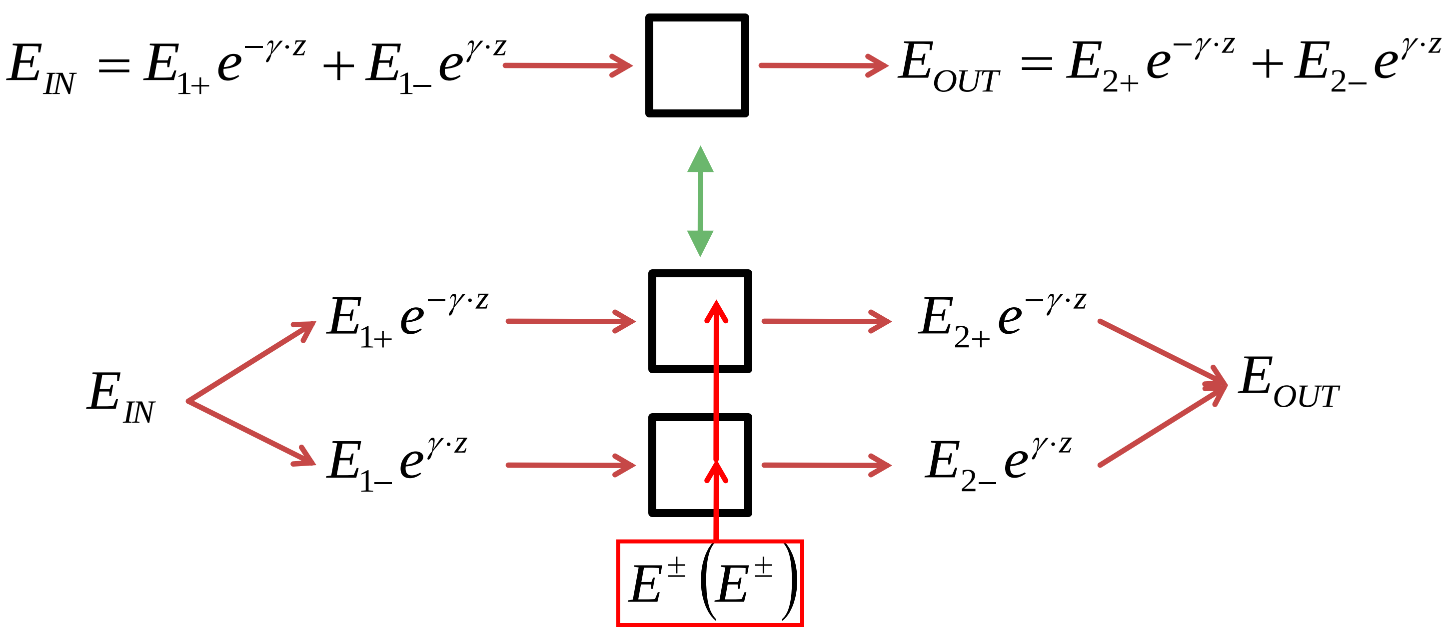


$$F(\omega)=\int_{-\infty}^{\infty} f(t)\cdot e^{-j\omega t}dt \qquad \boxed{G(\omega)[F(\omega)]} \qquad g(t)=\int_{-\infty}^{\infty} G(\omega)\cdot e^{j\omega t}d\omega$$

Mathematical modeling

- particular cases where analytical solution exists
  - wave in a single direction $E^{+}\left(E^{+}\right), \quad E^{-}\left(E^{-}\right)$
  - wave
    - incident
    - reflected
  - wave
    - direct
    - inverse

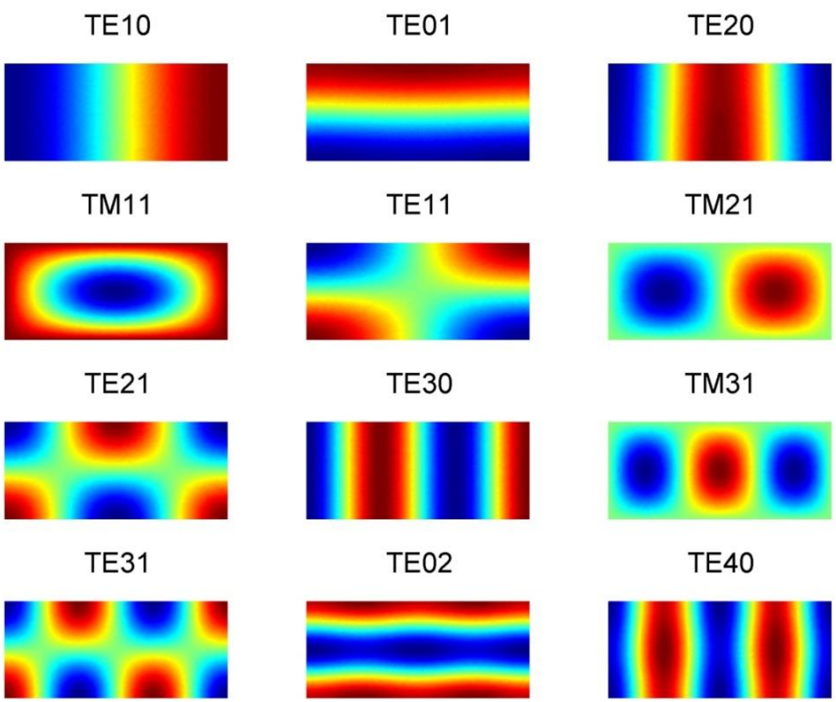
$$E_y = E^{+} \cdot e^{-\alpha \cdot z} \cdot e^{j(\omega t - \beta \cdot z)} + E^{-} \cdot e^{-\alpha \cdot z} \cdot e^{j(\omega t + \beta \cdot z)}$$

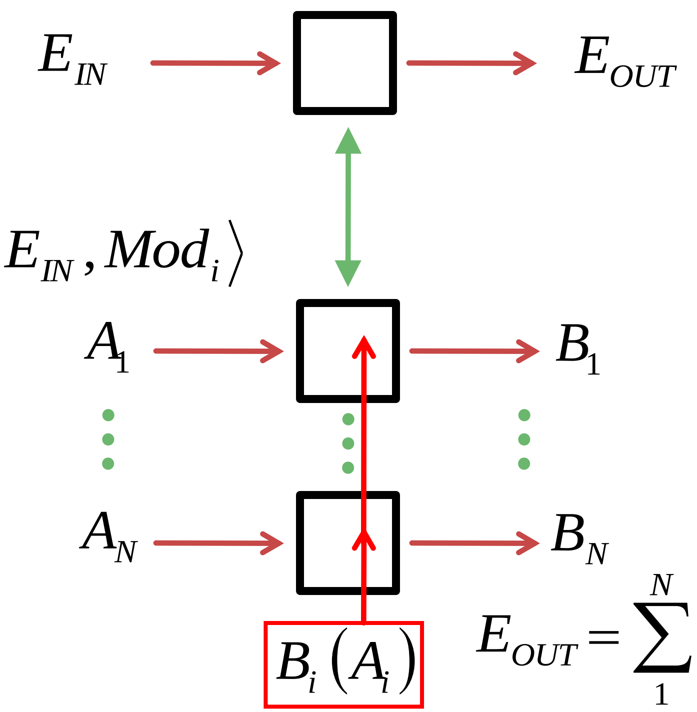


Mathematical modeling

- particular cases where analytical solution exists
  - modes in delimited media $B_i(A_i)$

$$E = \sum_{1}^{\infty} A_i \cdot Mod_i \qquad A_i = \langle E, Mod_i \rangle$$





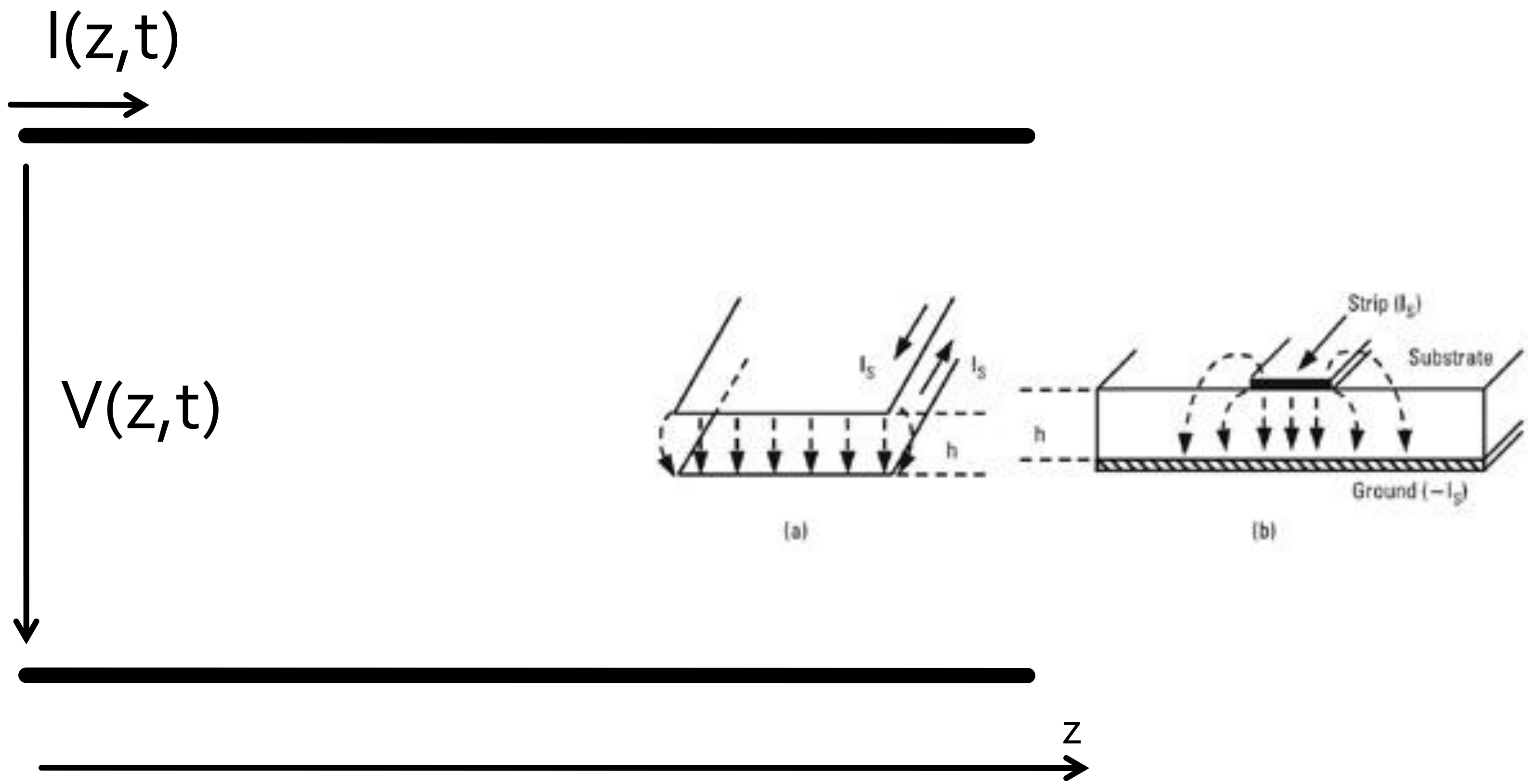
$$A_i = \langle E_{IN}, Mod_i \rangle$$

$$E_{OUT} = \sum_{1}^{N} B_i \cdot Mod_i$$

TEM transmission lines

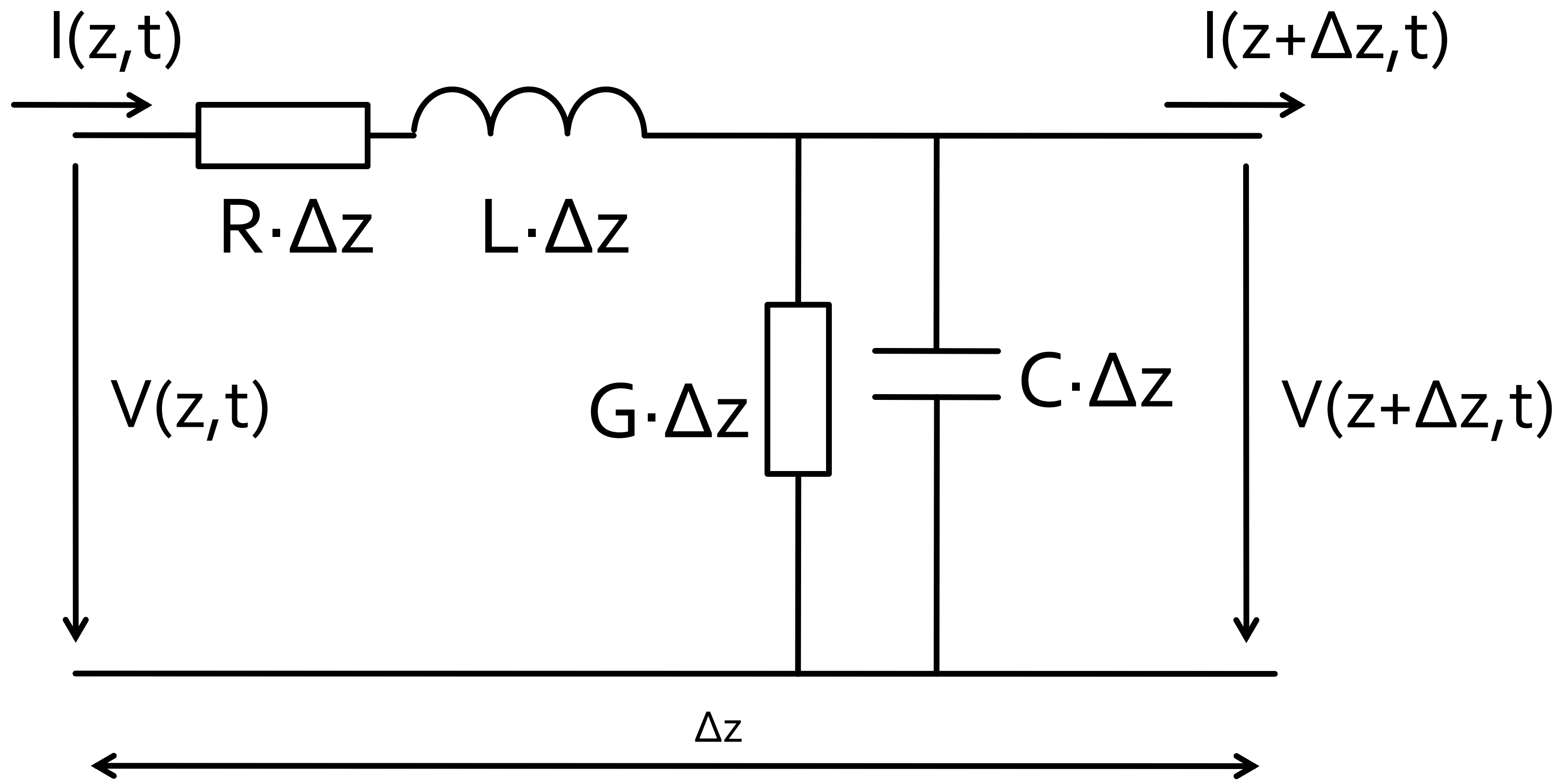
Transmission line

- TEM wave propagation, at least two conductors



Transmission line equivalent model

- TEM wave propagation, at least two conductors



Solutions

$$\begin{cases} V(z) = V_0^+ e^{-\gamma \cdot z} + V_0^- e^{\gamma \cdot z} \\ I(z) = I_0^+ e^{-\gamma \cdot z} + I_0^- e^{\gamma \cdot z} \end{cases}$$

$$\gamma = \alpha + j \cdot \beta = \sqrt{(R + j \cdot \omega \cdot L) \cdot (G + j \cdot \omega \cdot C)}$$

$$V(z) = V_0^+ e^{-\gamma \cdot z} + V_0^- e^{\gamma \cdot z}$$

$$\frac{dV(z)}{dz} = -(R + j \cdot \omega \cdot L) \cdot I(z)$$

$$I(z) = \frac{\gamma}{R + j \cdot \omega \cdot L} \left(V_0^+ e^{-\gamma \cdot z} - V_0^- e^{\gamma \cdot z} \right)$$

$$Z_0 \equiv \frac{R + j \cdot \omega \cdot L}{\gamma} = \sqrt{\frac{R + j \cdot \omega \cdot L}{G + j \cdot \omega \cdot C}}$$

- Characteristic impedance of the line

$$\frac{V_0^+}{I_0^+} = Z_0 = -\frac{V_0^-}{I_0^-}$$

$$\lambda = \frac{2\pi}{\beta} \qquad v_f = \frac{\omega}{\beta} = \lambda \cdot f$$

The lossless line

- **Lossless**: R=G=0

$$\gamma = \alpha + j\cdot\beta = \sqrt{(R + j\cdot\omega\cdot L)\cdot(G + j\cdot\omega\cdot C)} = j\cdot\omega\cdot\sqrt{L\cdot C}$$

$$\alpha = 0 \quad ; \quad \beta = \omega\cdot\sqrt{L\cdot C}$$

$$Z_0 = \sqrt{\frac{R + j\cdot\omega\cdot L}{G + j\cdot\omega\cdot C}} = \sqrt{\frac{L}{C}}$$

- Z_o is **real**

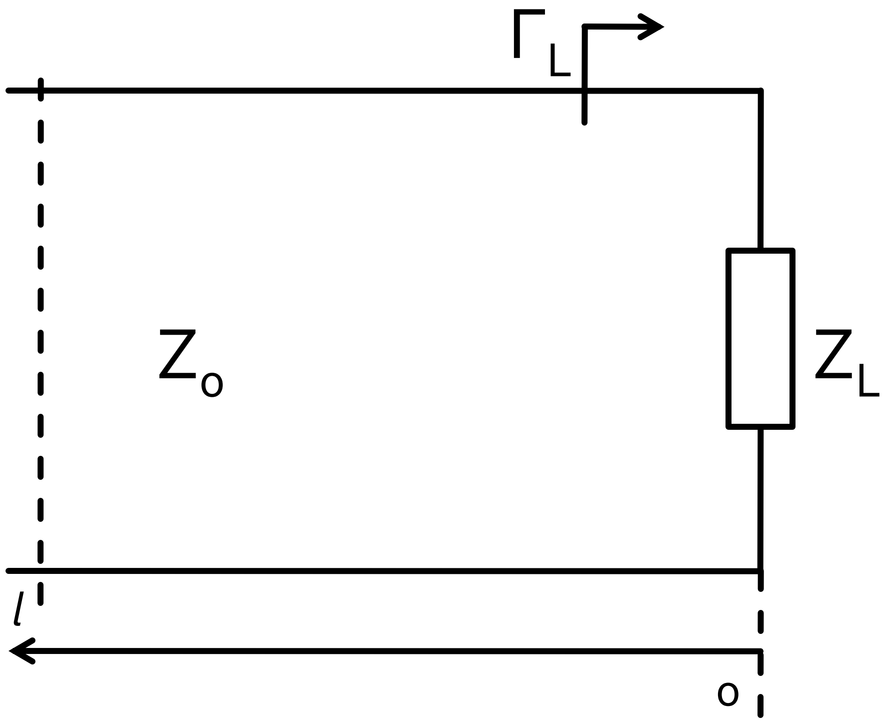
$$V(z) = V_0^+ e^{-j\cdot\beta\cdot z} + V_0^- e^{j\cdot\beta\cdot z}$$

$$I(z) = \frac{V_0^+}{Z_0} e^{-j\cdot\beta\cdot z} - \frac{V_0^-}{Z_0} e^{j\cdot\beta\cdot z}$$

$$\lambda = \frac{2\pi}{\omega\cdot\sqrt{LC}}$$

$$v_f = \frac{1}{\sqrt{LC}}$$

The lossless line



$$V(z) = V_0^+ e^{-j\cdot\beta\cdot z} + V_0^- e^{j\cdot\beta\cdot z}$$

$$I(z) = \frac{V_0^+}{Z_0} e^{-j\cdot\beta\cdot z} - \frac{V_0^-}{Z_0} e^{j\cdot\beta\cdot z}$$

$$Z_L = \frac{V(0)}{I(0)} \qquad Z_L = \frac{V_0^+ + V_0^-}{V_0^+ - V_0^-} \cdot Z_0$$

- voltage reflection coefficient

$$\Gamma = \frac{V_0^-}{V_0^+} = \frac{Z_L - Z_0}{Z_L + Z_0}$$

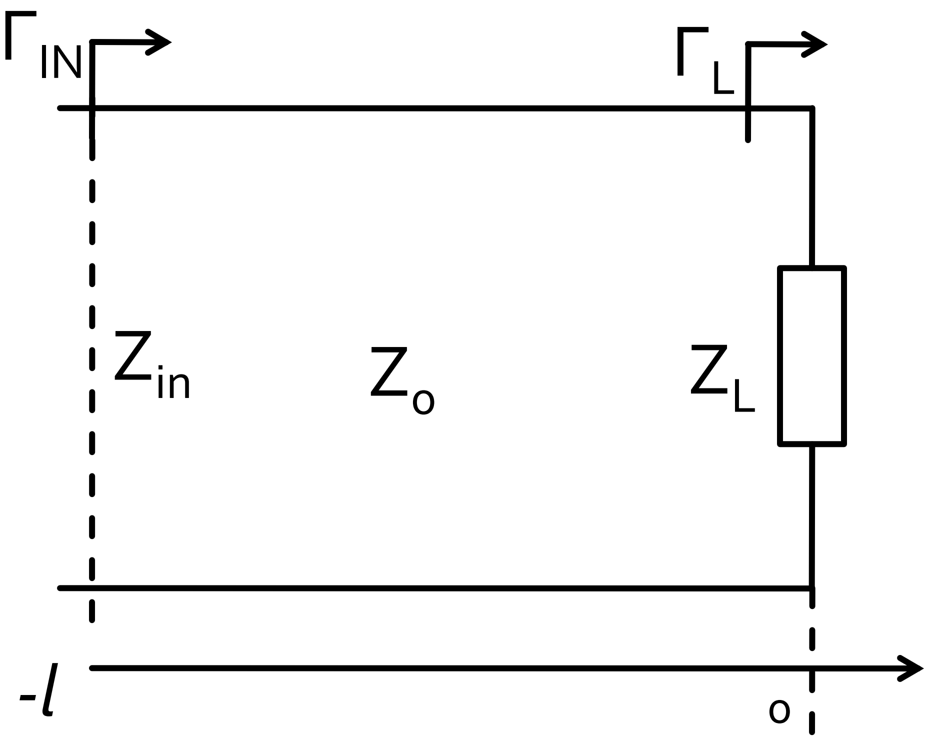
  - Z_o real

The lossless line

- voltage reflection coefficient seen at the input of the line

$$V(z) = V_0^+ e^{-j\cdot\beta\cdot z} + V_0^- e^{j\cdot\beta\cdot z}$$

$$\Gamma = \Gamma(z) = \frac{V_0^-(z)}{V_0^+(z)}$$



$$V(0) = V_0^+ + V_0^-$$

$$\Gamma(0) = \Gamma_L = \frac{V_0^-}{V_0^+}$$

$$V(-l) = V_0^+ e^{j\cdot\beta\cdot l} + V_0^- e^{-j\cdot\beta\cdot l}$$

$$\Gamma(-l) = \Gamma_{IN} = \frac{V_0^- \cdot e^{-j\cdot\beta\cdot l}}{V_0^+ \cdot e^{j\cdot\beta\cdot l}} = \Gamma(0)\cdot e^{-2j\cdot\beta\cdot l}$$

$$|\Gamma(-l)| = |\Gamma(0)|\cdot\left|e^{-2j\cdot\beta\cdot l}\right| = |\Gamma(0)|$$

$$\boxed{\Gamma_{IN} = \Gamma_L \cdot e^{-2j\cdot\beta\cdot l}} \qquad \boxed{|\Gamma_{IN}| = |\Gamma_L|}$$

The lossless line

$$V(z)=V_0^+\cdot\left(e^{-j\cdot\beta\cdot z}+\Gamma\cdot e^{j\cdot\beta\cdot z}\right) \qquad I(z)=\frac{V_0^+}{Z_0}\cdot\left(e^{-j\cdot\beta\cdot z}-\Gamma\cdot e^{j\cdot\beta\cdot z}\right)$$

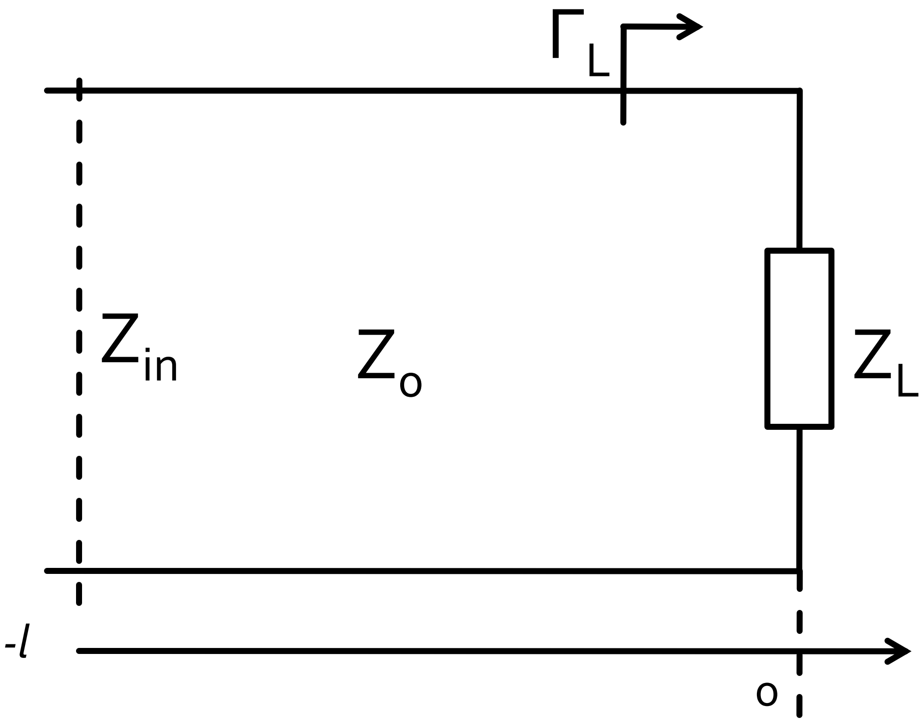
- time-average Power flow along the line

$$P_{avg}=\frac{1}{2}\cdot\mathrm{Re}\left\{V(z)\cdot I(z)^*\right\}=\frac{1}{2}\cdot\frac{\left|V_0^+\right|^2}{Z_0}\cdot\mathrm{Re}\left\{1-\Gamma^*\cdot\underbrace{e^{-2j\cdot\beta\cdot z}+\Gamma\cdot e^{2j\cdot\beta\cdot z}}_{\left(z-z^*\right)=\mathrm{Im}}-|\Gamma|^2\right\}$$

$$P_{avg}=\frac{1}{2}\cdot\frac{\left|V_0^+\right|^2}{Z_0}\cdot\left(1-|\Gamma|^2\right)$$

- Total power delivered to the load = Incident power – “Reflected” power
- Return “Loss” [dB] $\quad \mathrm{RL}=-20\cdot\log|\Gamma| \quad [\mathrm{dB}]$

The lossless line



$$V(-l) = V_0^+ e^{j\cdot\beta\cdot l} + V_0^- e^{-j\cdot\beta\cdot l}$$

$$I(-l) = \frac{V_0^+}{Z_0} e^{j\cdot\beta\cdot l} - \frac{V_0^-}{Z_0} e^{-j\cdot\beta\cdot l}$$

$$Z_{in} = \frac{V(-l)}{I(-l)} \qquad Z_{in} = Z_0 \cdot \frac{1+\Gamma\cdot e^{-2j\cdot\beta\cdot l}}{1-\Gamma\cdot e^{-2j\cdot\beta\cdot l}}$$

- the **input impedance** seen looking toward the load

$$Z_{in} = Z_0 \cdot \frac{(Z_L+Z_0)\cdot e^{j\cdot\beta\cdot l} + (Z_L-Z_0)\cdot e^{-j\cdot\beta\cdot l}}{(Z_L+Z_0)\cdot e^{j\cdot\beta\cdot l} - (Z_L-Z_0)\cdot e^{-j\cdot\beta\cdot l}}$$

$$Z_{in} = Z_0 \cdot \frac{Z_L + j\cdot Z_0\cdot\tan(\beta\cdot l)}{Z_0 + j\cdot Z_L\cdot\tan(\beta\cdot l)}$$

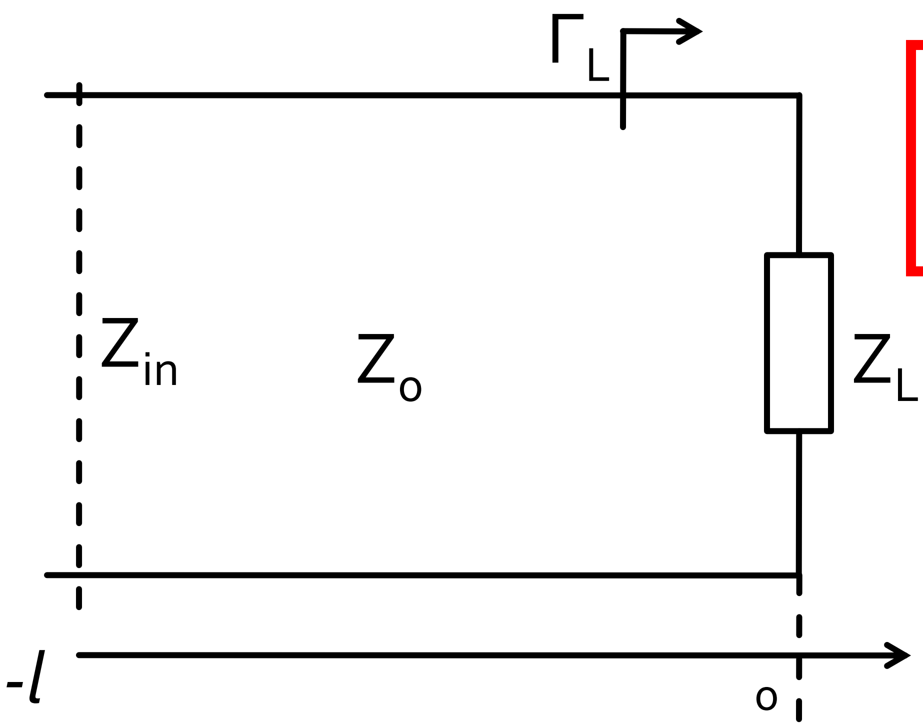
The lossless line

- the **input impedance** seen looking toward the load

$$\boxed{Z_{in} = Z_0 \cdot \frac{Z_L + j \cdot Z_0 \cdot \tan\beta \cdot l}{Z_0 + j \cdot Z_L \cdot \tan\beta \cdot l}}$$

The lossless line

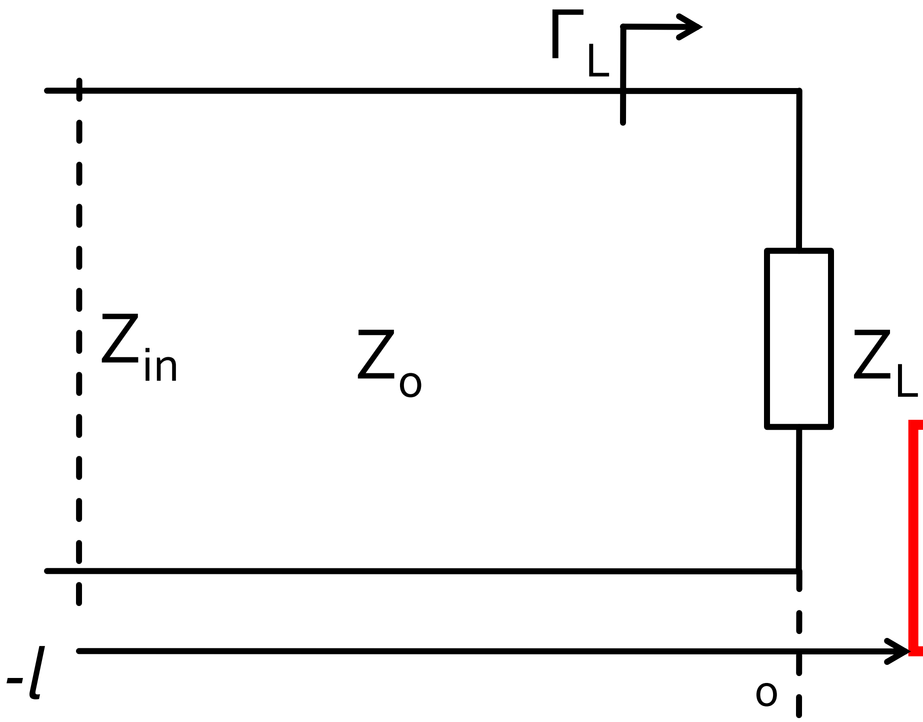
- input impedance of a length ***l*** of transmission line with characteristic impedance $\boldsymbol{Z_o}$, loaded with an arbitrary impedance $\boldsymbol{Z_L}$



$$\boxed{Z_{in} = Z_0 \cdot \frac{Z_L + j \cdot Z_0 \cdot \tan \beta \cdot l}{Z_0 + j \cdot Z_L \cdot \tan \beta \cdot l}}$$

The lossless line

- input impedance is **frequency dependent** through $\beta \cdot l$



$$v_f = \frac{\omega}{\beta} = \lambda \cdot f \qquad \lambda = \frac{2\pi}{\beta}$$

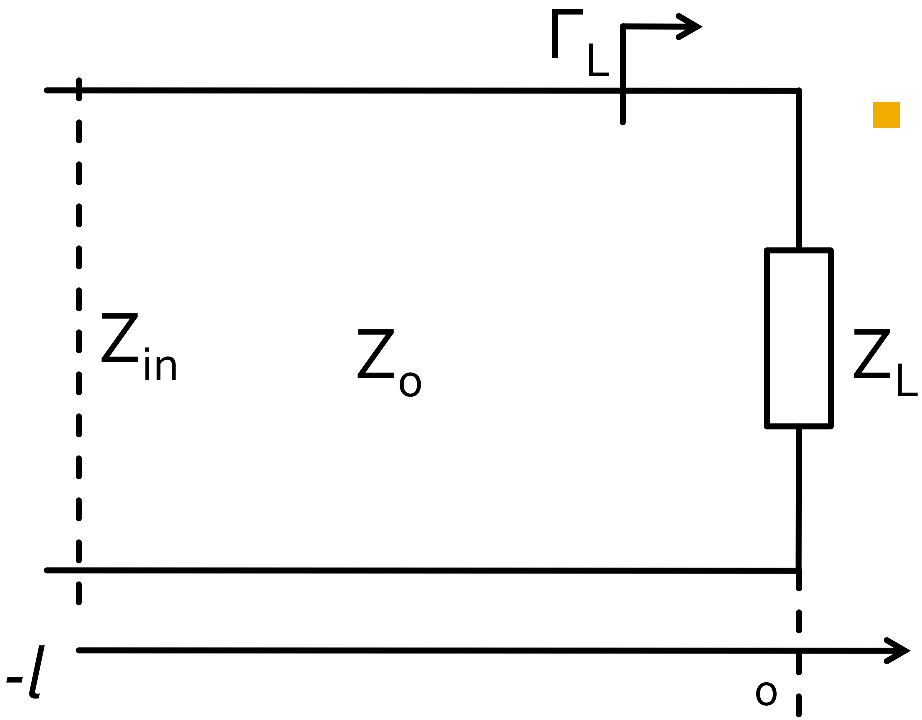
$$\beta \cdot l = \frac{2\pi}{\lambda} \cdot l = \frac{2\pi \cdot f}{v_f} \cdot l = \frac{2\pi \cdot l}{v_f} \cdot f$$

frequency dependence is periodical, imposed by the tan trigonometric function

$$\boxed{Z_{in} = Z_0 \cdot \frac{Z_L + j \cdot Z_0 \cdot \tan \beta \cdot l}{Z_0 + j \cdot Z_L \cdot \tan \beta \cdot l}}$$

The lossless line, special cases

- $l = k \cdot \lambda/2$ $\quad \beta \cdot l = \frac{2\pi}{\lambda} \cdot l = k \cdot \pi \quad \tan \beta \cdot l = 0 \quad \boxed{Z_{in} = Z_0}$
- $l = \lambda/4 + k \cdot \lambda/2$ $\quad \tan \beta \cdot l \to \infty \quad \boxed{Z_{in} = \frac{Z_0^2}{Z_L}}$



- quarter-wave transformer

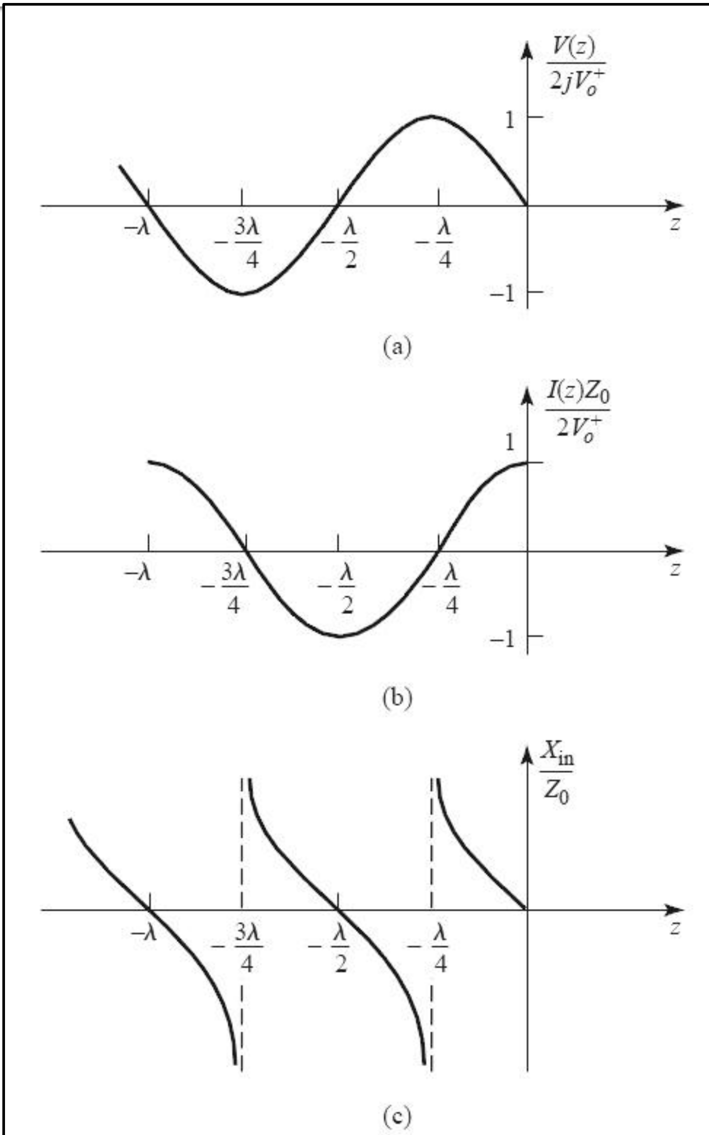
$$\boxed{Z_{in} = Z_0 \cdot \frac{Z_L + j \cdot Z_0 \cdot \tan \beta \cdot l}{Z_0 + j \cdot Z_L \cdot \tan \beta \cdot l}}$$

Short-circuited transmission line

- $Z_L = 0$
- purely imaginary for any length *l*
  - +/- → depending on *l* value

$$Z_{in} = j \cdot Z_0 \cdot \tan \beta \cdot l$$

$$Z_{in} = Z_0 \cdot \frac{Z_L + j \cdot Z_0 \cdot \tan(\beta \cdot l)}{Z_0 + j \cdot Z_L \cdot \tan(\beta \cdot l)}$$

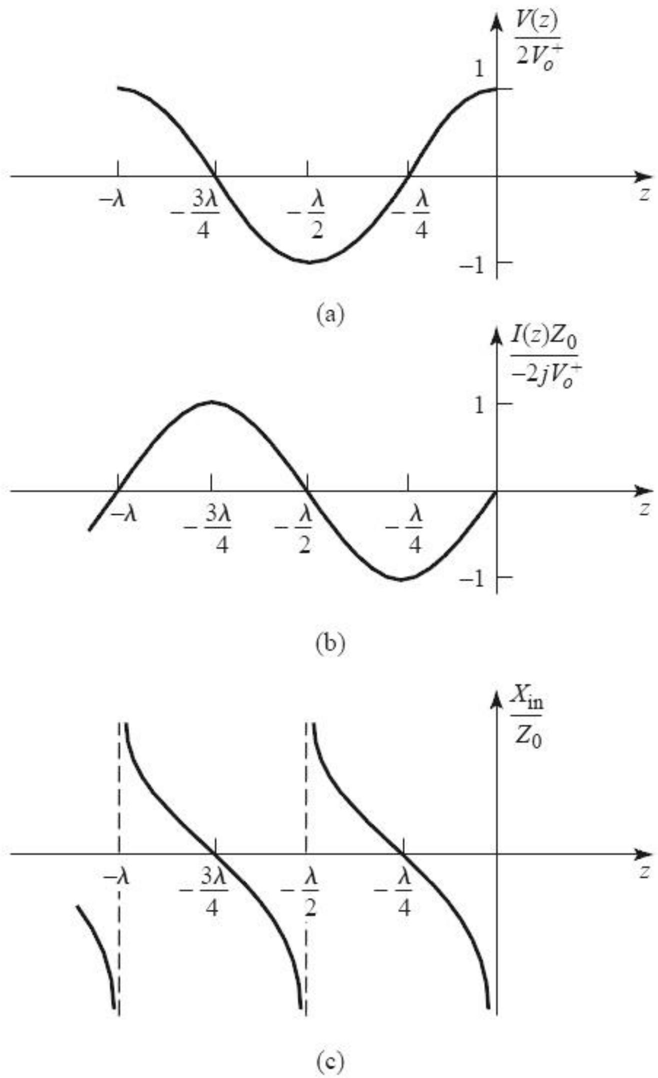


Open-circuited transmission line

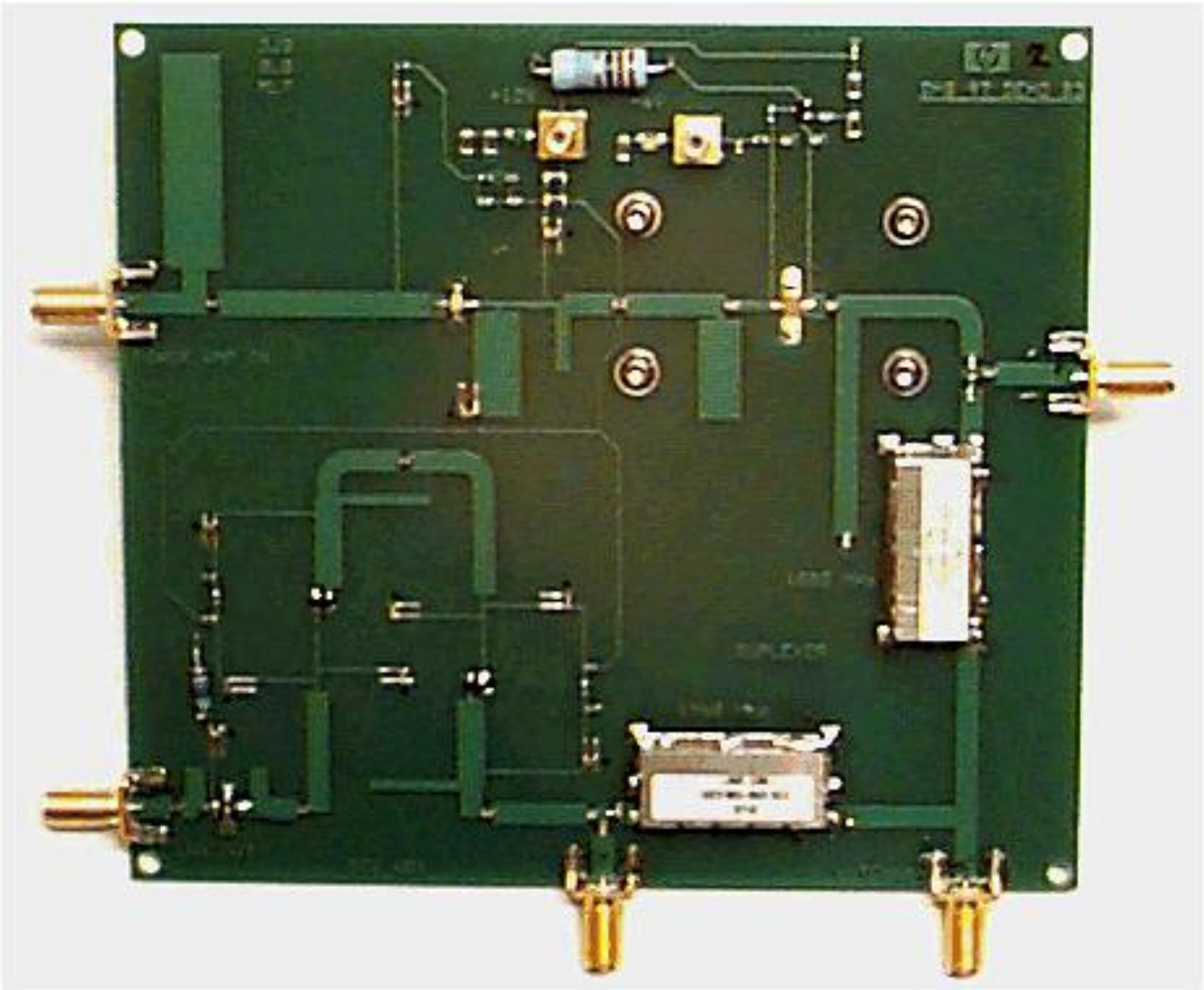
- $Z_L = \infty \longrightarrow 1 / Z_L = 0$
- purely imaginary for any length *l*
  - +/- → depending on *l* value

$$Z_{in} = -j \cdot Z_0 \cdot \cot \beta \cdot l$$

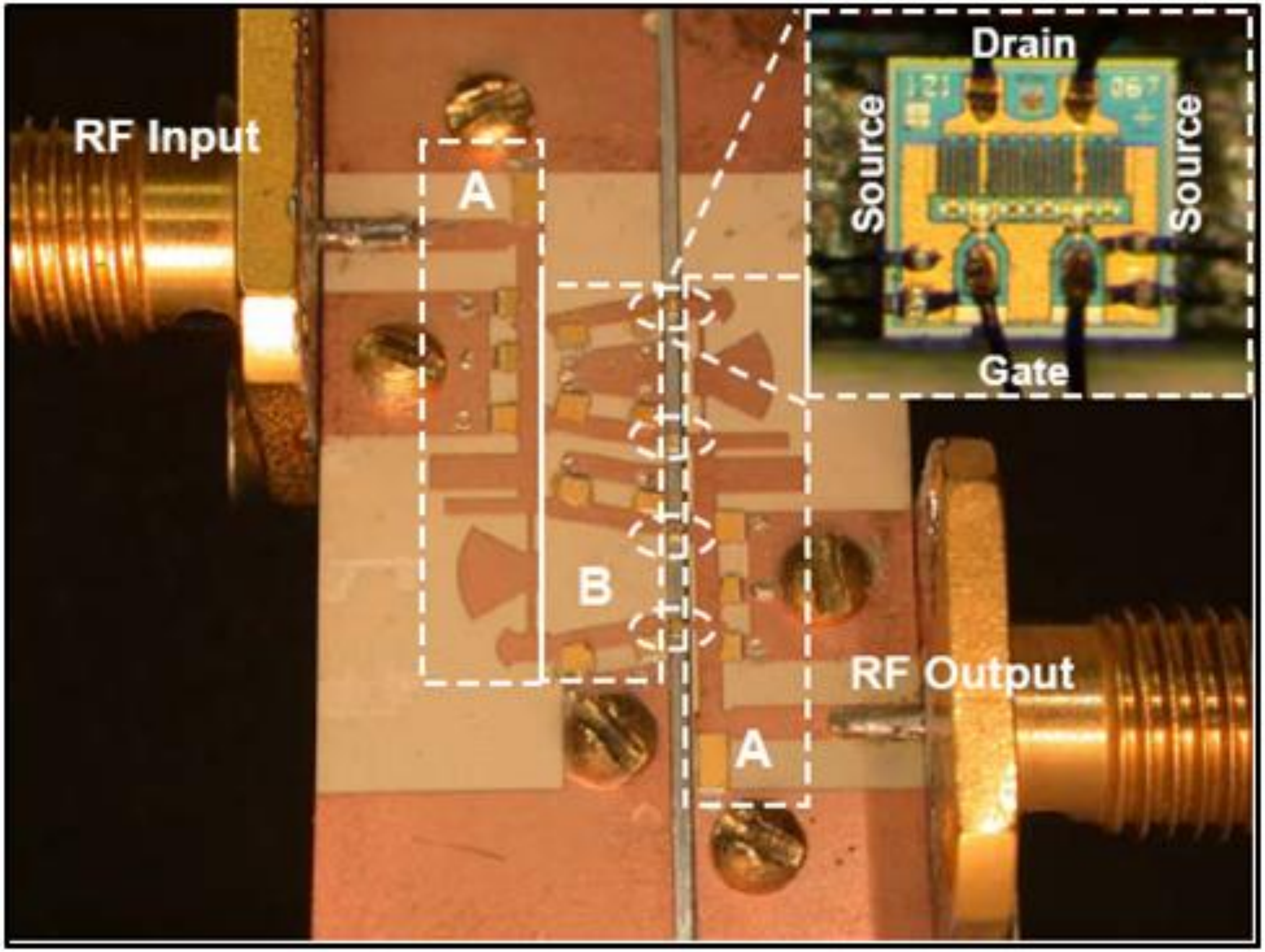
$$Z_{in} = Z_0 \cdot \frac{Z_L + j \cdot Z_0 \cdot \tan(\beta \cdot l)}{Z_0 + j \cdot Z_L \cdot \tan(\beta \cdot l)}$$



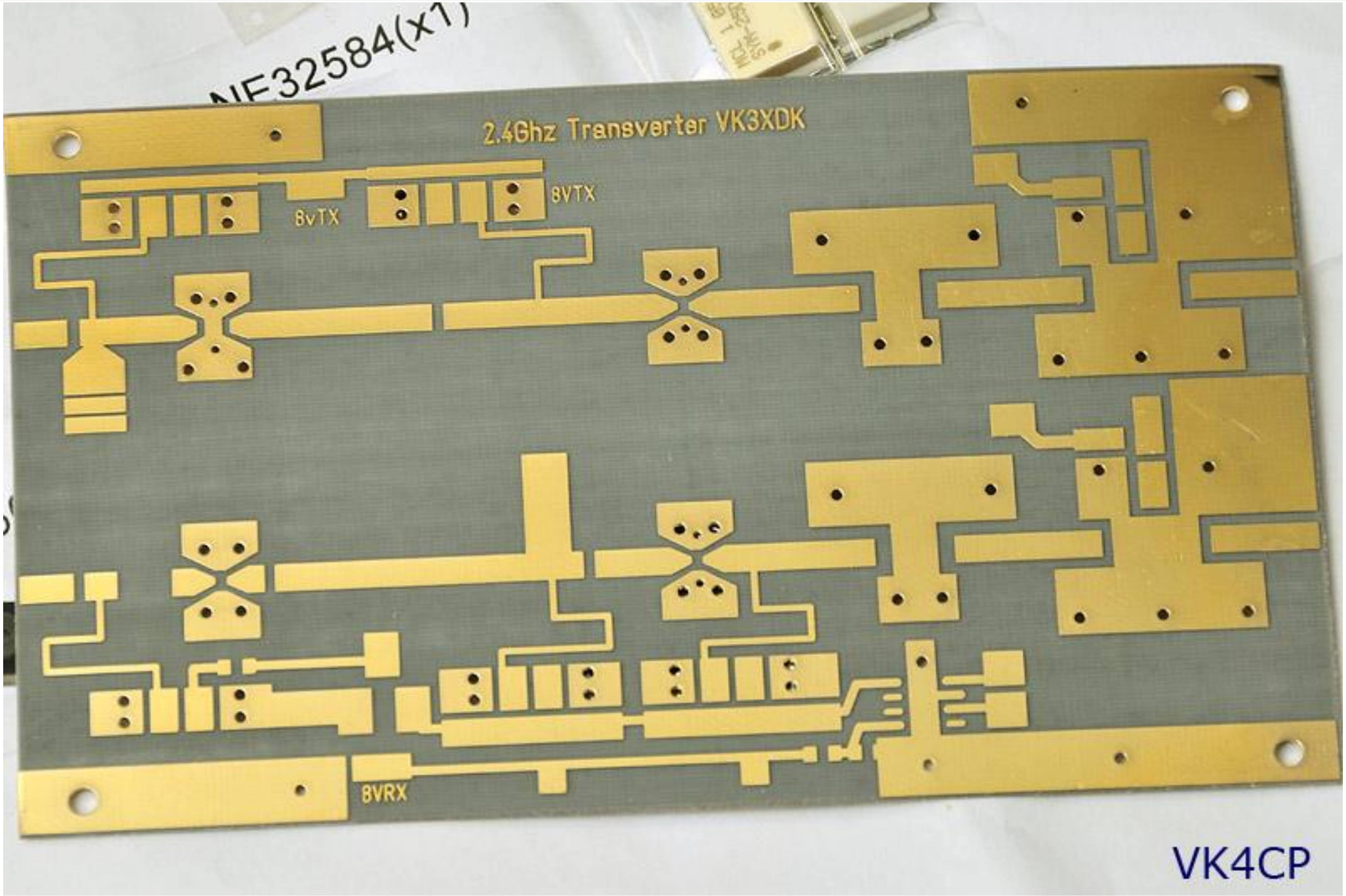
Examples



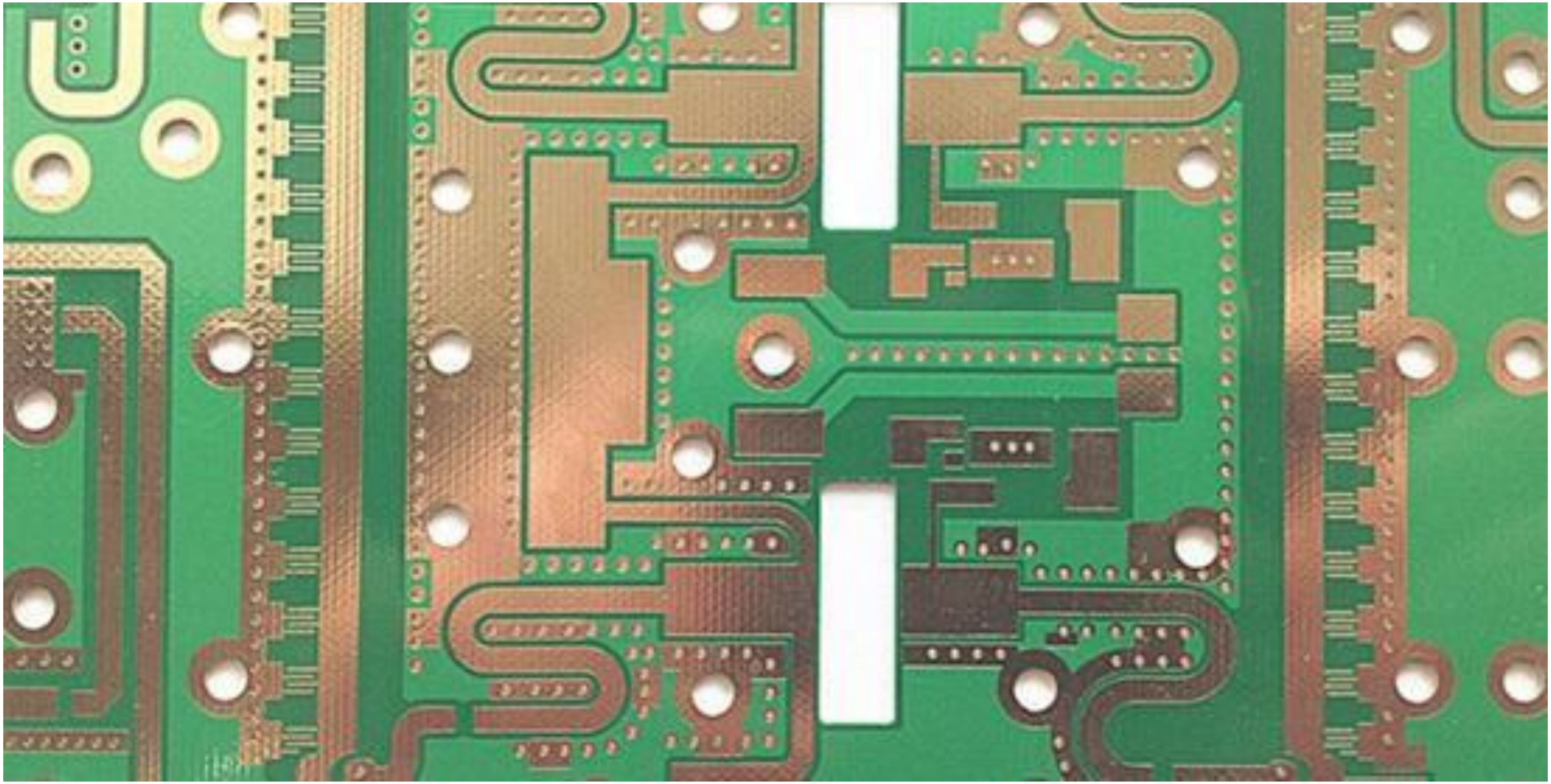
Examples



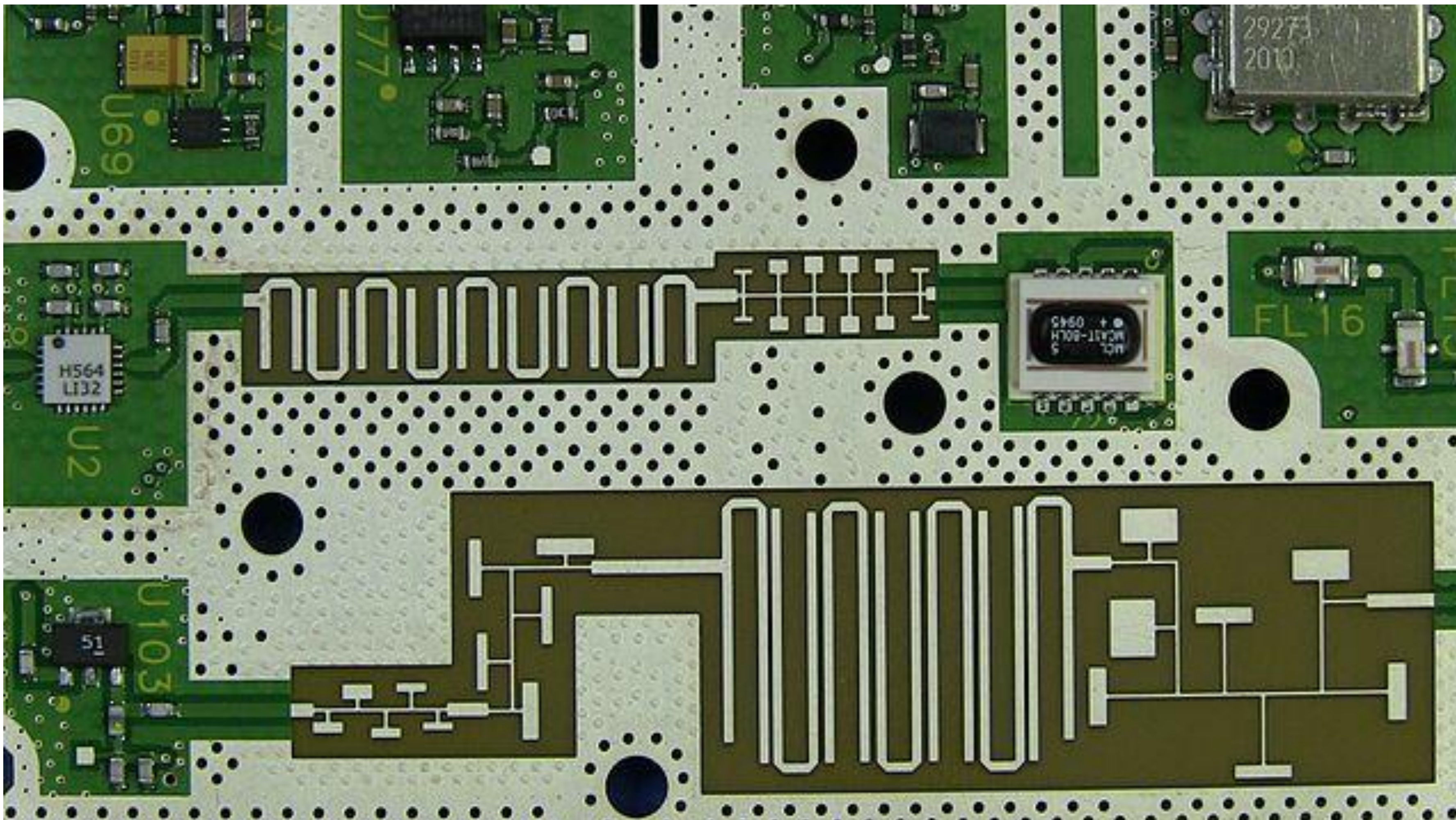
Examples



Examples



Examples



Voltage standing wave ratio

$$V(z)=V_0^+\cdot\left(e^{-j\cdot\beta\cdot z}+\Gamma\cdot e^{j\cdot\beta\cdot z}\right)\qquad |V(z)|=\left|V_0^+\right|\cdot\left|e^{-j\cdot\beta\cdot z}\right|\cdot\left|1+\Gamma\cdot e^{2j\cdot\beta\cdot z}\right|\qquad \Gamma=|\Gamma|\cdot e^{j\theta}$$

$$|V(z)|=\left|V_0^+\right|\cdot\left|1+|\Gamma|\cdot e^{\theta+2j\cdot\beta\cdot z}\right|$$

maximum magnitude value for $e^{\theta+2j\cdot\beta\cdot z}=1$ $\qquad V_{\max}=\left|V_0^+\right|\cdot\left(1+|\Gamma|\right)$

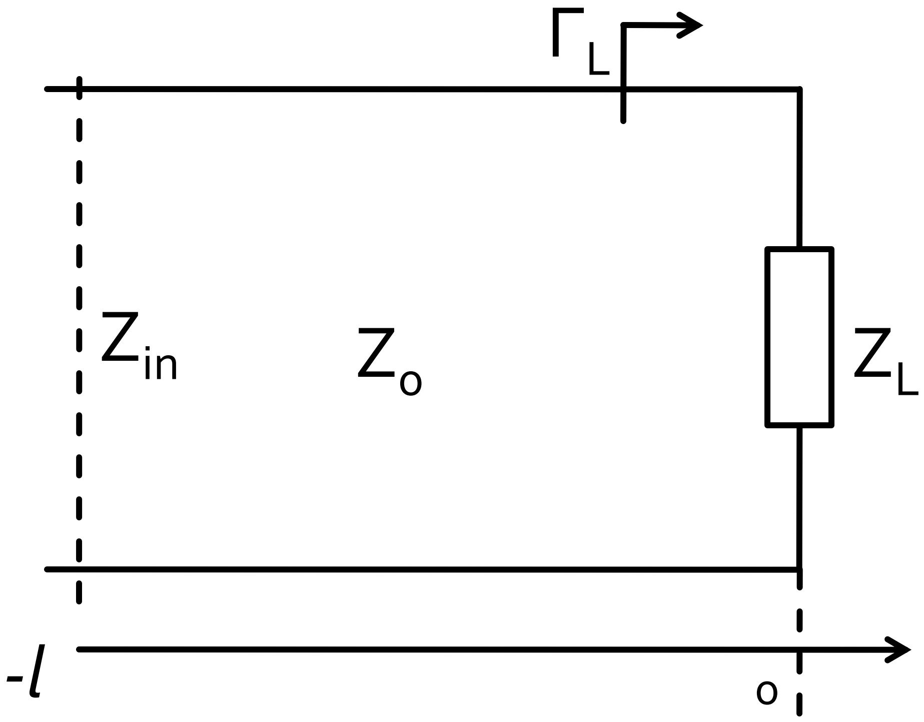
minimum magnitude value for $e^{\theta+2j\cdot\beta\cdot z}=-1$ $\qquad V_{\min}=\left|V_0^+\right|\cdot\left(1-|\Gamma|\right)$

- SWR is defined as the ratio between maximum and minimum
  - (Voltage) Standing Wave Ratio

$$VSWR=\frac{V_{\max}}{V_{\min}}=\frac{1+|\Gamma|}{1-|\Gamma|}$$

  - real number $1 \le VSWR < \infty$
  - a measure of the mismatch (SWR = 1 means a matched line)

The lossless line +/-



$$Z_{in} = Z_0 \cdot \frac{Z_L + j \cdot Z_0 \cdot \tan \beta \cdot l}{Z_0 + j \cdot Z_L \cdot \tan \beta \cdot l}$$

$$V(z) = V_0^+ e^{-\gamma \cdot z} + V_0^- e^{\gamma \cdot z}$$

$$I(z) = I_0^+ e^{-\gamma \cdot z} + I_0^- e^{\gamma \cdot z}$$

$$\Gamma(-l) = \Gamma(0) \cdot e^{-2 j \cdot \beta \cdot l}$$

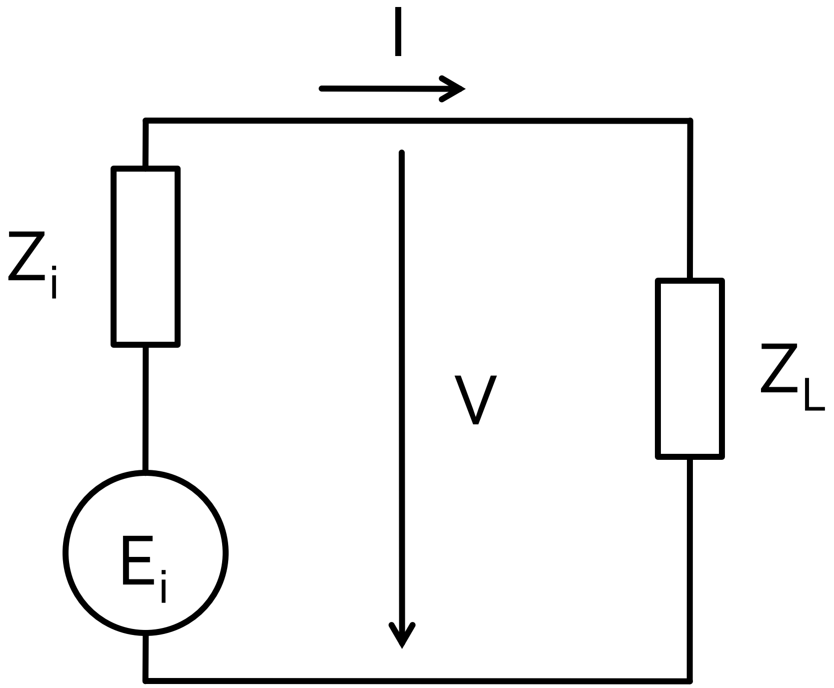
$$\Gamma_{in} = \Gamma_L \cdot e^{-2 j \cdot \beta \cdot l}$$

Impedance Matching with Impedance Transformers (Lab 1)

Impedance Matching

Matching

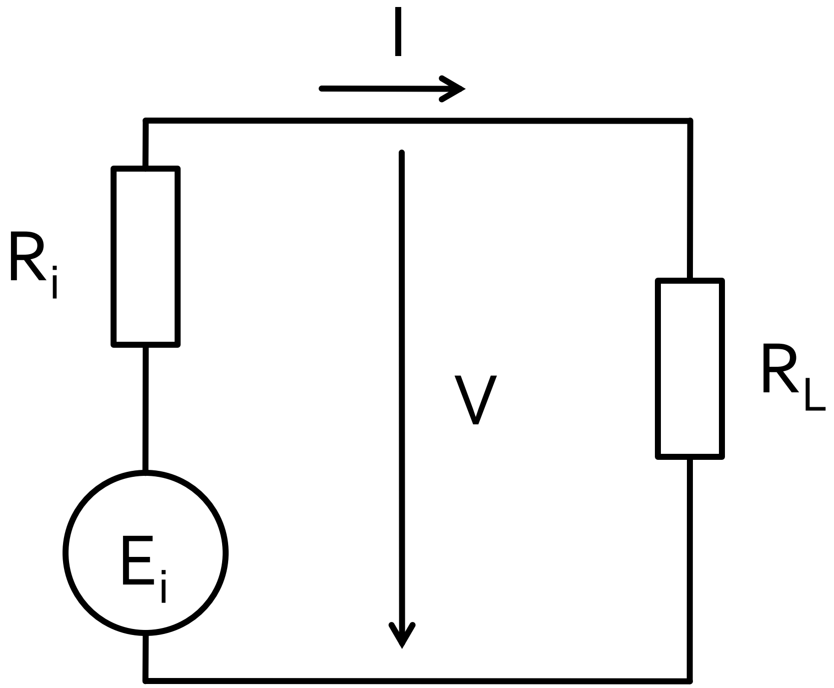
- Source **matched** to load ?



- impedance values ?
- existence of reflections ?

Matching, real impedances

- Source matched to load



$$I = \frac{E_i}{R_i + R_L}$$

$$V = \frac{E_i \cdot R_L}{R_i + R_L}$$

$$P_L = R_L \cdot I^2$$

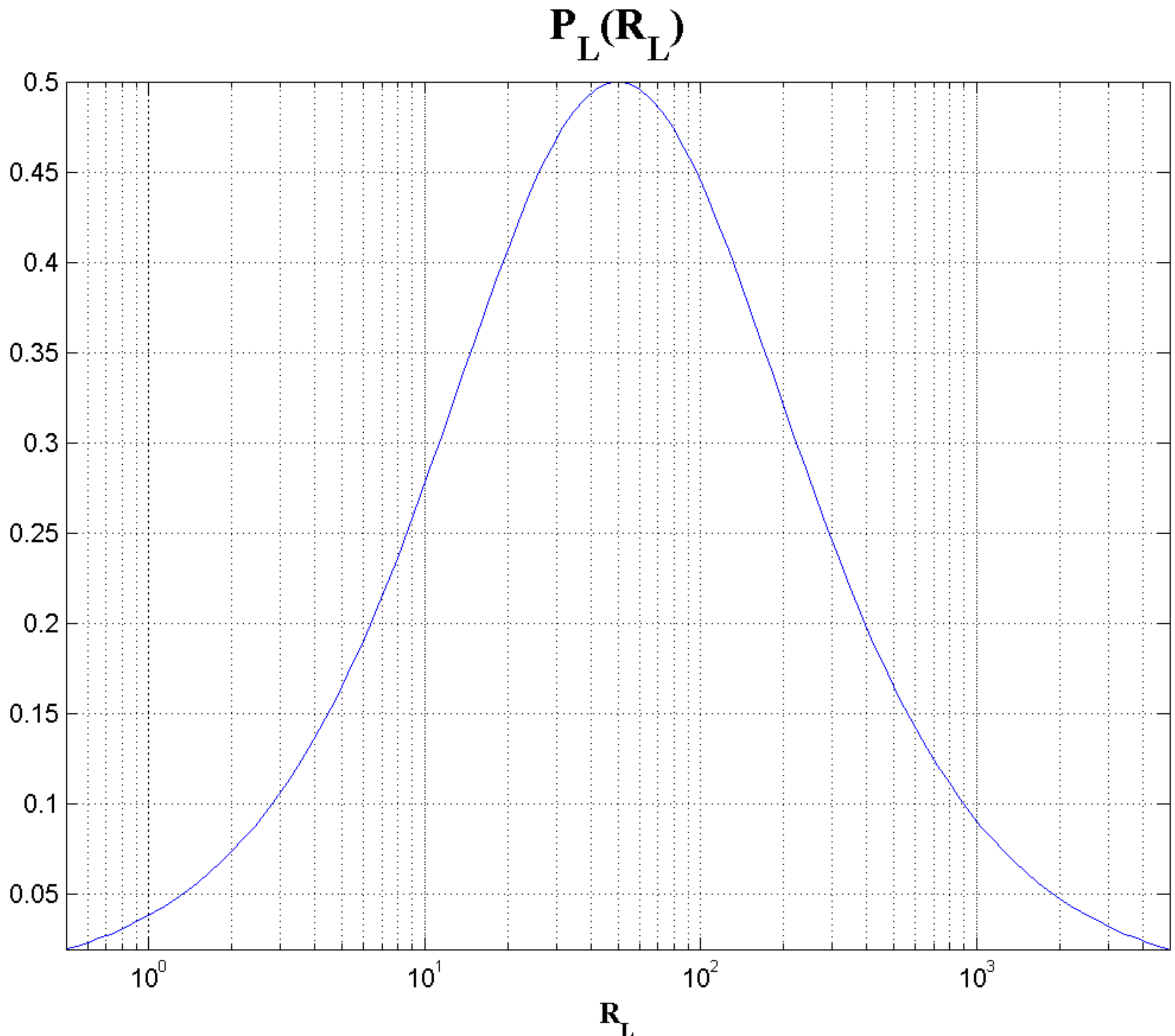
$$P_L = \frac{R_L \cdot E_i^2}{(R_i + R_L)^2}$$

Matching, real impedances

$$P_L = R_L \cdot I^2 \qquad P_L = \frac{R_L \cdot E_i^2}{(R_i + R_L)^2}$$

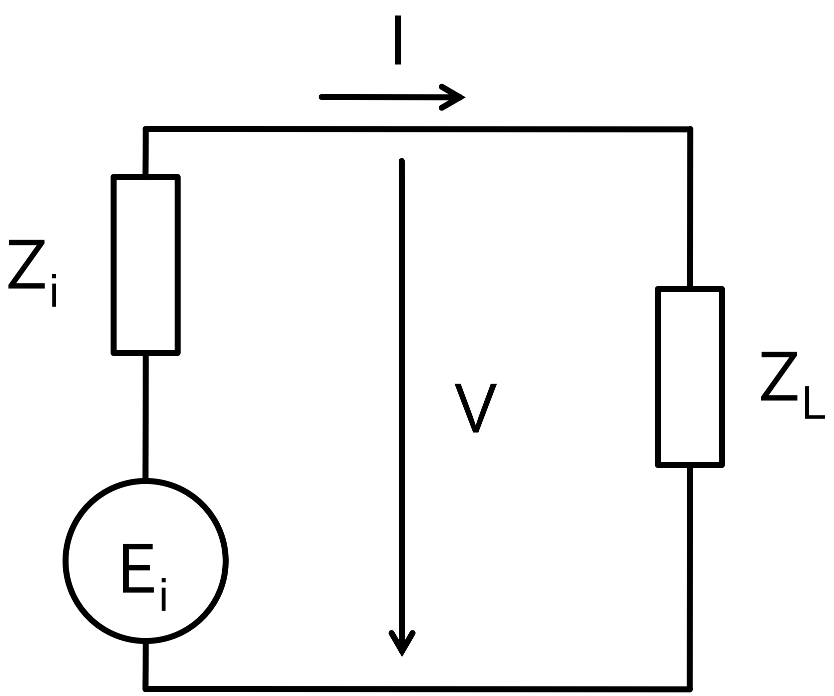
- Power dissipated on load
  - $R_i = 50\Omega$
  - $R_L = 0 \rightarrow P_L = 0$
  - $R_L = \infty \rightarrow P_L = 0$

Matching, real impedances



Matching, complex impedances

- Source matched to load



$$I = \frac{E_i}{Z_i + Z_L}$$

$$V = \frac{E_i \cdot Z_L}{Z_i + Z_L}$$

$$P_L = \mathrm{Re}\left\{ Z_L \cdot \left| I \right|^2 \right\}$$

$$P_L = \mathrm{Re}\left\{ Z_L \right\} \cdot \left| \frac{E_i}{Z_i + Z_L} \right|^2$$

Matching

$$P_L = \frac{R_L \cdot |E_i|^2}{|Z_i + Z_L|^2} = \frac{R_L \cdot |E_i|^2}{|(R_i + R_L) + j \cdot (X_i + X_L)|^2}$$

$$|a + j \cdot b| = \sqrt{a^2 + b^2}$$

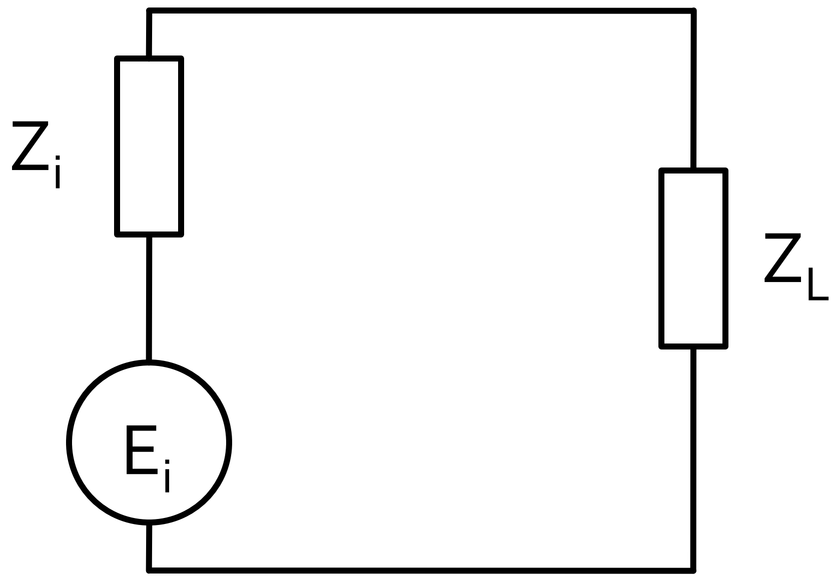
$$P_L = \frac{R_L \cdot |E_i|^2}{(R_i + R_L)^2 + (X_i + X_L)^2}$$

- Matching
  - maximum power transmitted to the load
  - condition?

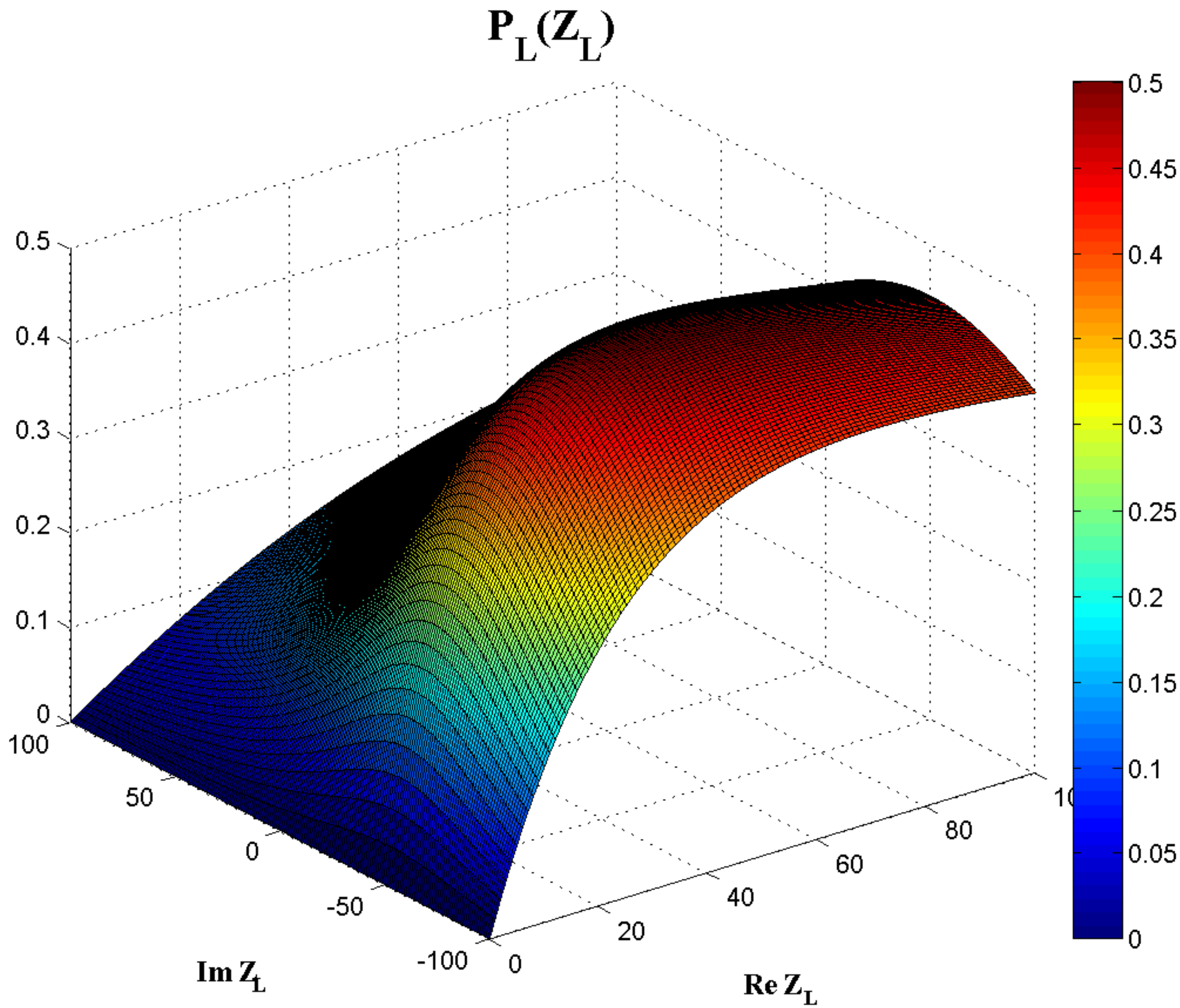
Matching, example

- $E = 10V$
- $Z_i = 50\,\Omega + j\cdot 50\Omega$
- $P_L(Z_L)$?

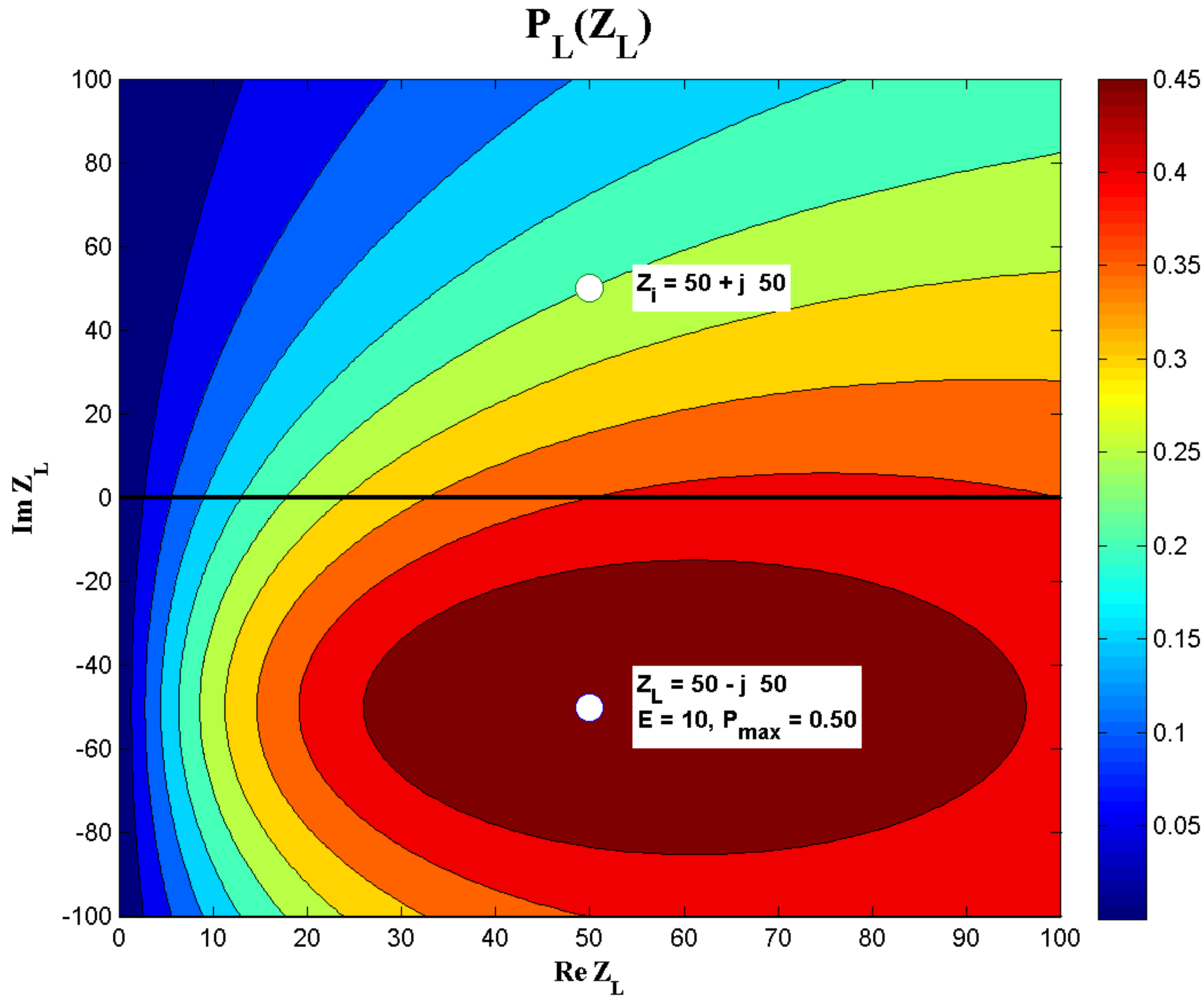
$$P_L = \frac{R_L \cdot |E_i|^2}{(R_i + R_L)^2 + (X_i + X_L)^2}$$



Matching, example



Matching, example



Matching , from the point of view of power transmission

$$R_i > 0,\ R_L > 0 \qquad P_L = \frac{|E_i|^2}{4R_i + \frac{(R_i - R_L)^2}{R_L} + \frac{(X_i + X_L)^2}{R_L}}$$

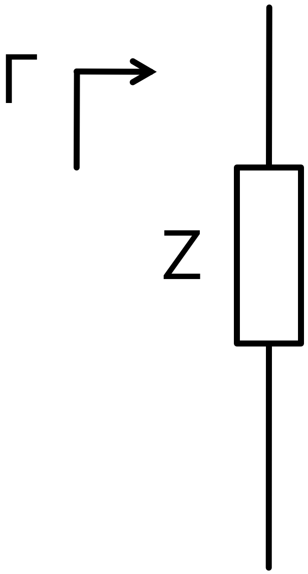
$$P_{L\max} = \frac{|E_i|^2}{4R_i} \equiv P_a \qquad R_L = R_i,\ X_L = -X_i$$

- P_a : Available Power

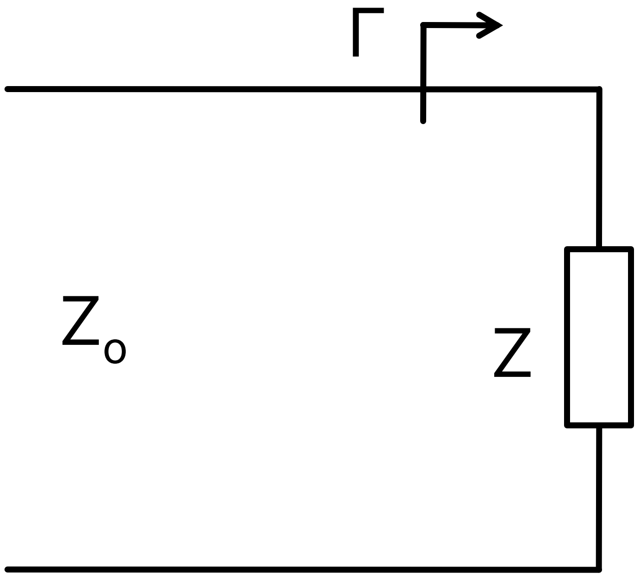
$$Z_L = Z_i^*$$

Reflection coefficient

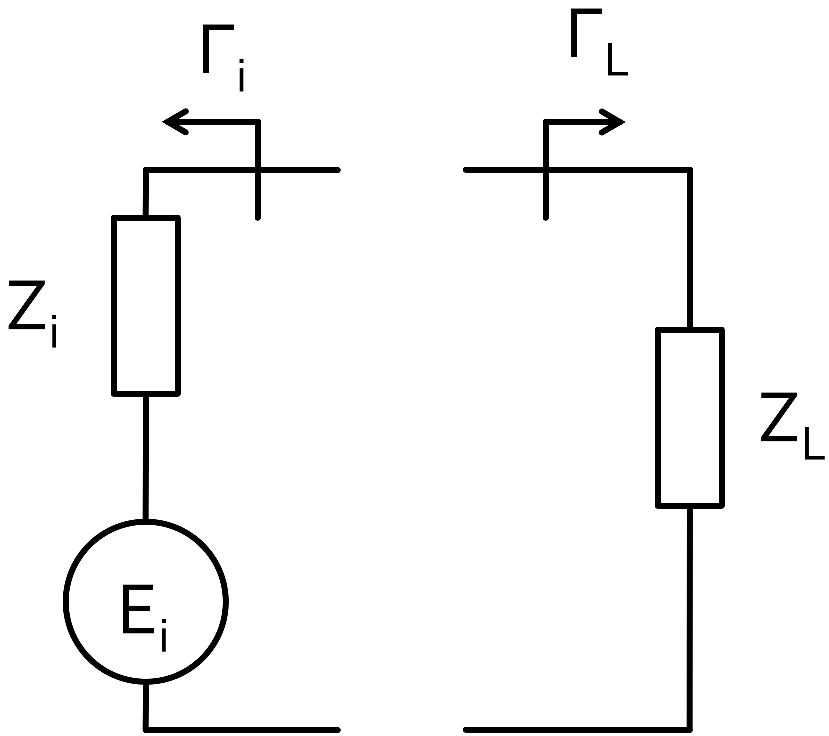
- Any impedance Z_o chosen as reference



$$\Gamma = \frac{Z - Z_0^*}{Z + Z_0}$$



Matching , from the point of view of power transmission



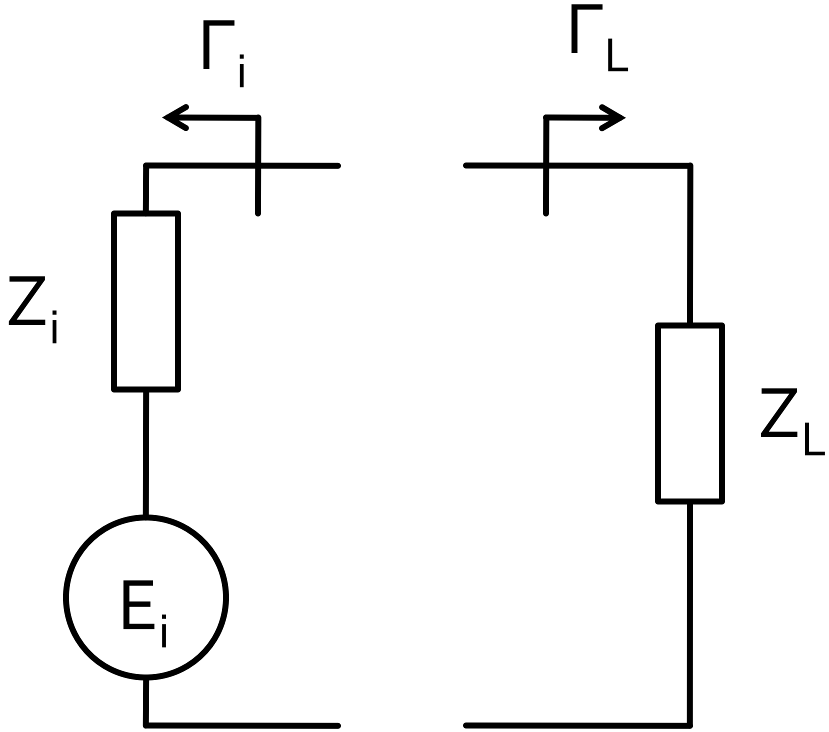
$$\Gamma_i = \frac{Z_i - Z_0^*}{Z_i + Z_0}$$

$$\Gamma_i = \frac{(R_i - R_0) + j \cdot (X_i + X_0)}{(R_i + R_0) + j \cdot (X_i + X_0)}$$

$$\Gamma_L = \frac{Z_L - Z_0^*}{Z_L + Z_0}$$

$$\Gamma_L = \frac{(R_L - R_0) + j \cdot (X_L + X_0)}{(R_L + R_0) + j \cdot (X_L + X_0)}$$

Matching , from the point of view of power transmission



$$\Gamma_i = \frac{Z_i - Z_0^*}{Z_i + Z_0} = 1 - \frac{Z_0 + Z_0^*}{Z_i + Z_0}$$

$$\Gamma_L = \frac{Z_L - Z_0^*}{Z_L + Z_0} = 1 - \frac{Z_0 + Z_0^*}{Z_L + Z_0}$$

$$\Gamma_i^* = 1 - \frac{Z_0^* + Z_0}{Z_i^* + Z_0^*} = 1 - \frac{Z_0^* + Z_0}{Z_L + Z_0^*}$$

Matching , from the point of view of power transmission

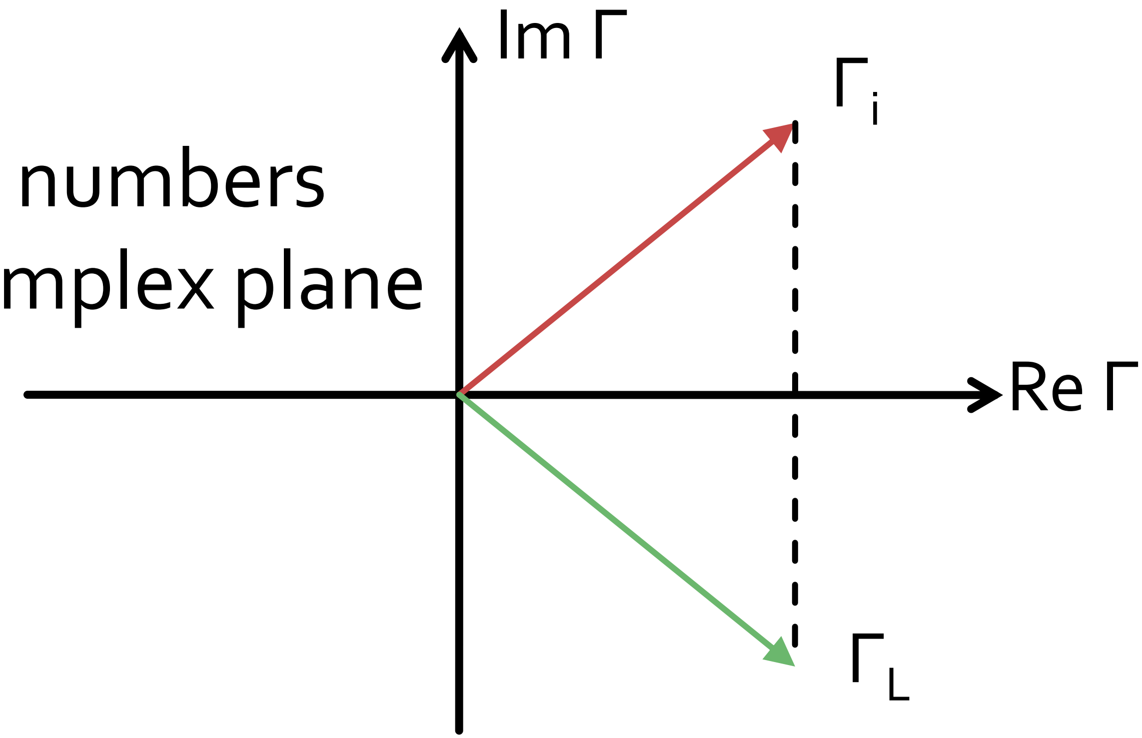
$$Z_L = Z_i^*$$

If we choose a real Zo

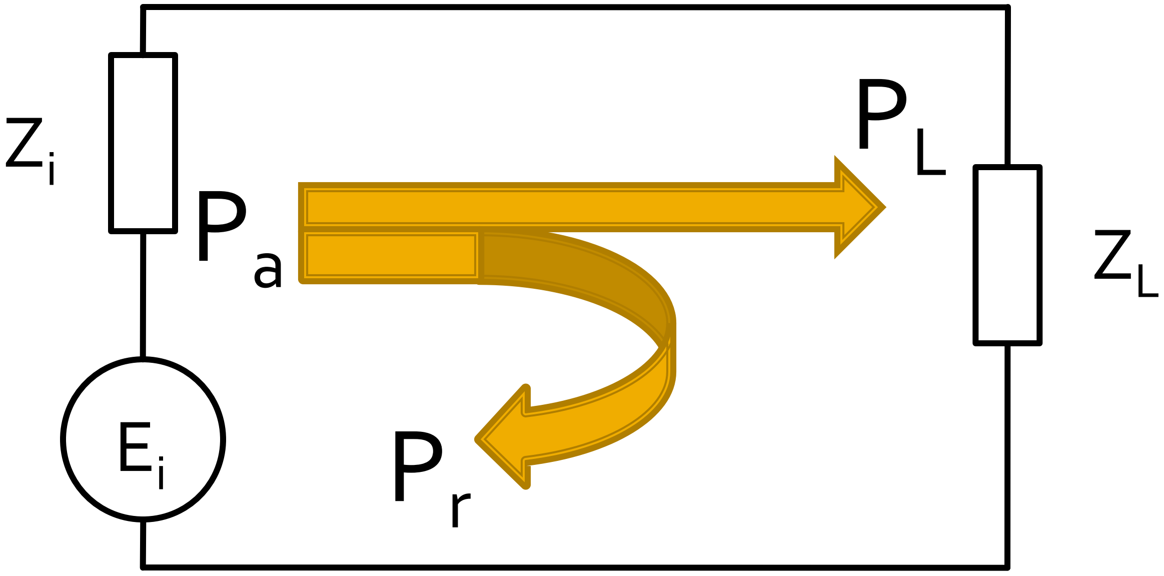
$$\Gamma = \frac{Z - Z_0}{Z + Z_0}$$

$$\Gamma_L = \Gamma_i^*$$

- complex numbers
- in the complex plane



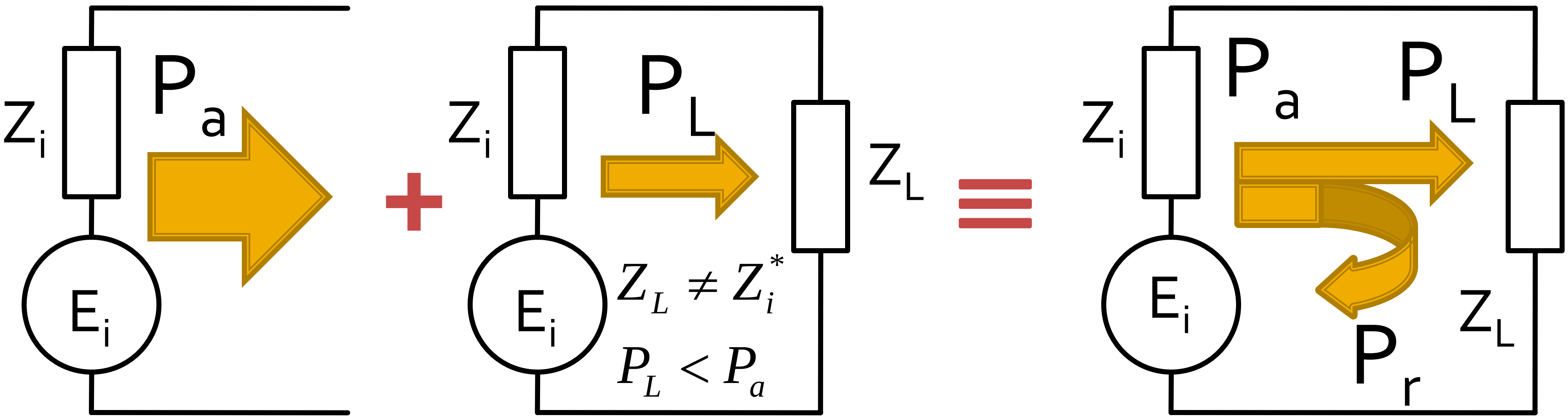
Reflection and power / Model



$$Z_L \neq Z_i^*$$

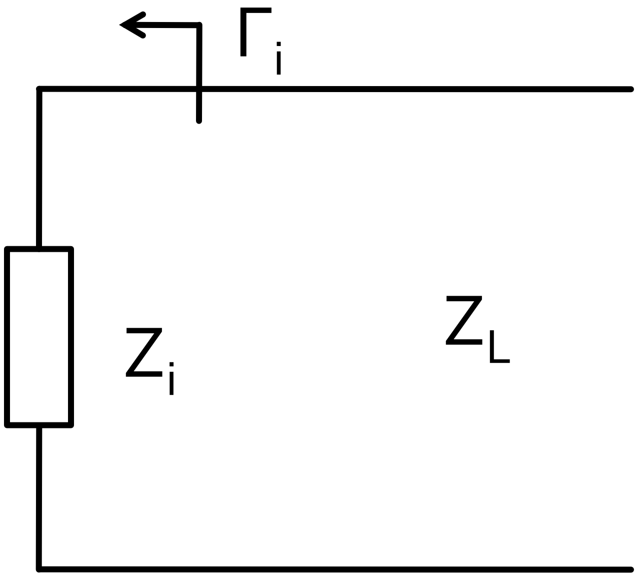
- ~~Power reflection~~
- Power of the reflected wave

Reflection and power / Model

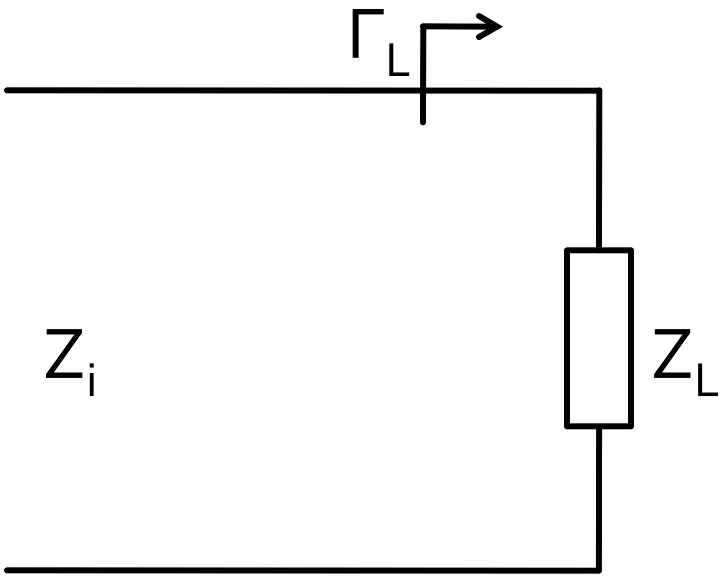


- The source has the ability to sent to the load a certain maximum power (available power) P_a
- For a particular load the power sent to the load is less than the maximum (mismatch) $P_L < P_a$
- The phenomenon is **"as if"** (model) some of the power is reflected $P_r = P_a - P_L$
- The power is a **scalar !**

Reflection coefficient



$$\Gamma_i = \frac{Z_i - Z_L^*}{Z_i + Z_L}$$



$$\Gamma_L = \frac{Z_L - Z_i^*}{Z_L + Z_i}$$

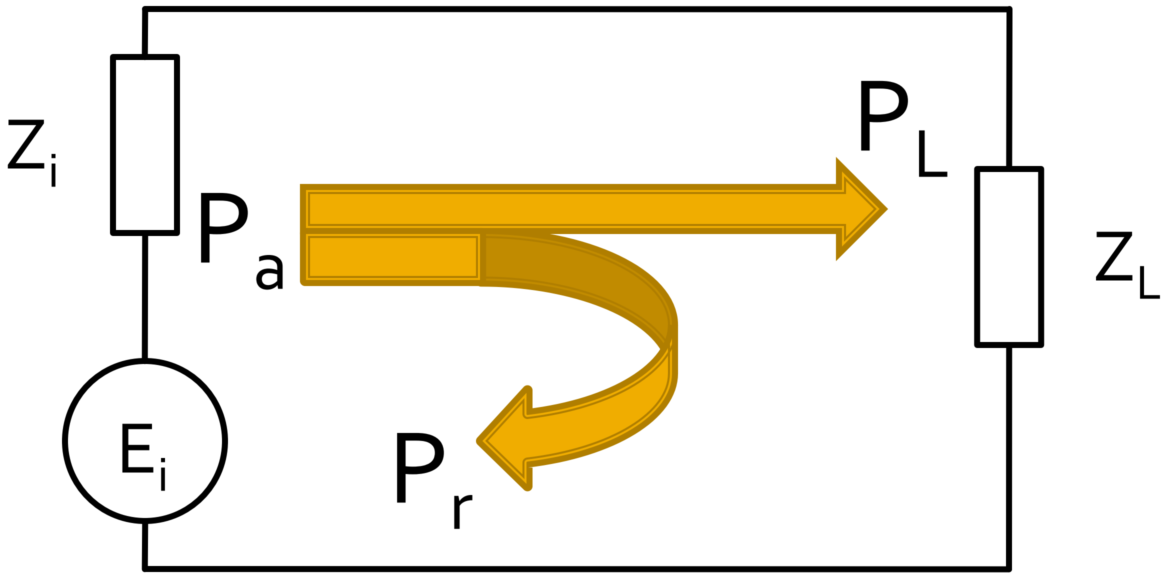
Reflection coefficient

$$\Gamma_i = \frac{(R_i - R_L) + j\cdot(X_i + X_L)}{(R_i + R_L) + j\cdot(X_i + X_L)} \qquad \Gamma_L = \frac{(R_L - R_i) + j\cdot(X_L + X_i)}{(R_L + R_i) + j\cdot(X_L + X_i)}$$

$$|\Gamma_i| = \frac{|(R_i - R_L) + j\cdot(X_i + X_L)|}{|(R_i + R_L) + j\cdot(X_i + X_L)|} = \frac{\sqrt{(R_i - R_L)^2 + (X_i + X_L)^2}}{\sqrt{(R_i + R_L)^2 + (X_i + X_L)^2}} = |\Gamma_L|$$

$$|\Gamma_i| = |\Gamma_L| \equiv |\Gamma|$$

Reflection and power / Model



$$P_a = \frac{|E_i|^2}{4R_i}$$

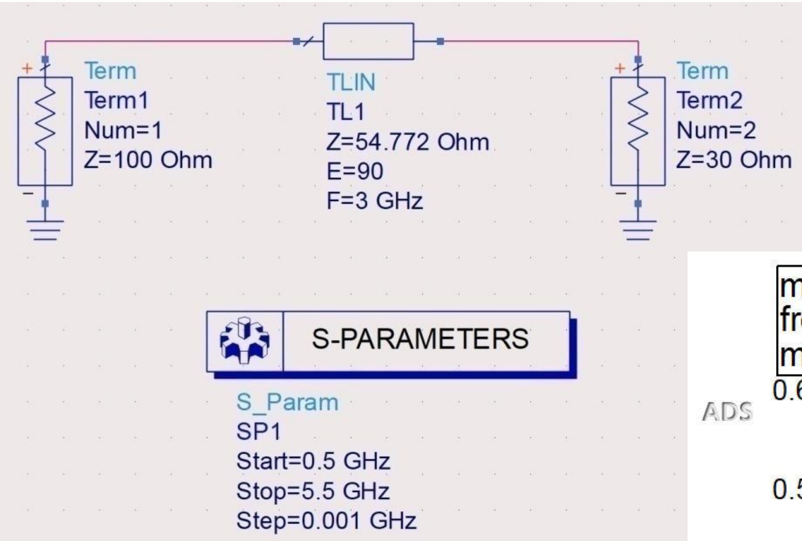
$$P_L = \frac{R_L \cdot |E_i|^2}{(R_i+R_L)^2 + (X_i+X_L)^2}$$

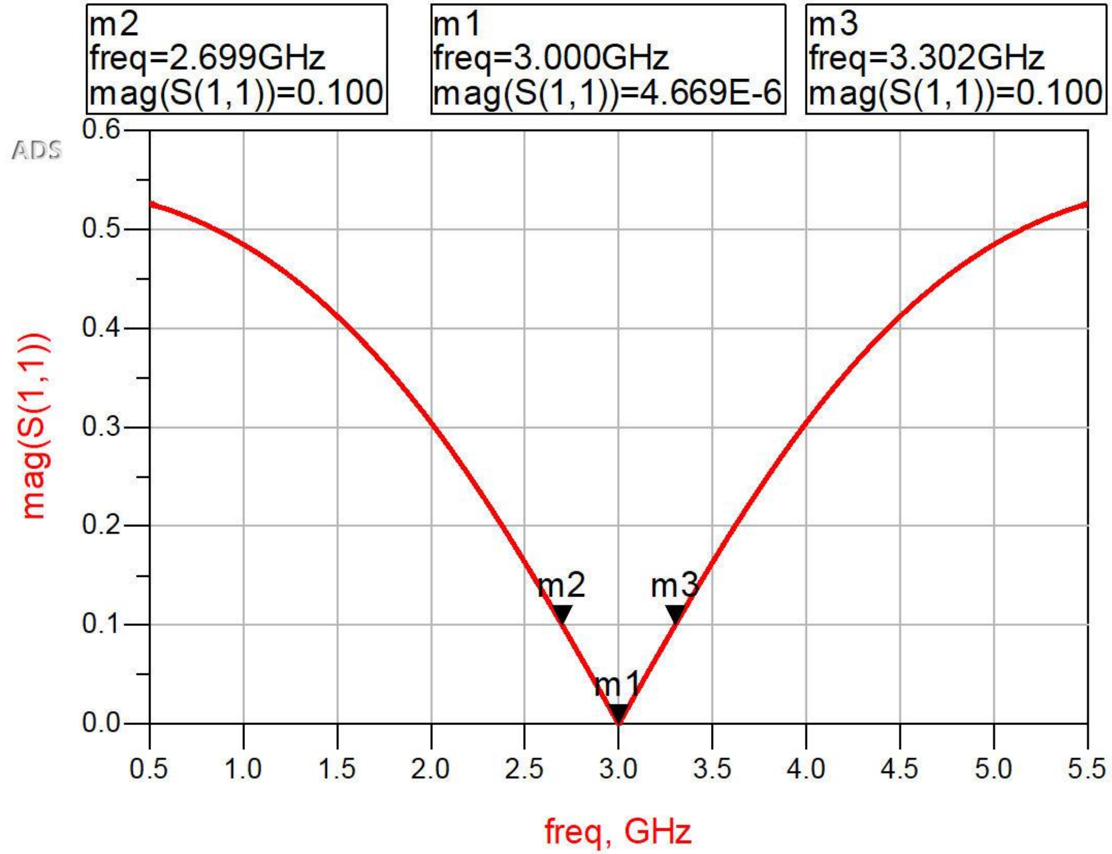
$$P_r = P_a - P_L = \frac{|E_i|^2}{4R_i} - \frac{R_L \cdot |E_i|^2}{(R_i+R_L)^2 + (X_i+X_L)^2} = \frac{|E_i|^2}{4R_i} \cdot \left[1 - \frac{4R_L \cdot R_i}{(R_i+R_L)^2 + (X_i+X_L)^2}\right]$$

$$P_r = \frac{|E_i|^2}{4R_i} \cdot \left[\frac{(R_i-R_L)^2 + (X_i+X_L)^2}{(R_i+R_L)^2 + (X_i+X_L)^2}\right] = P_a \cdot |\Gamma|^2$$

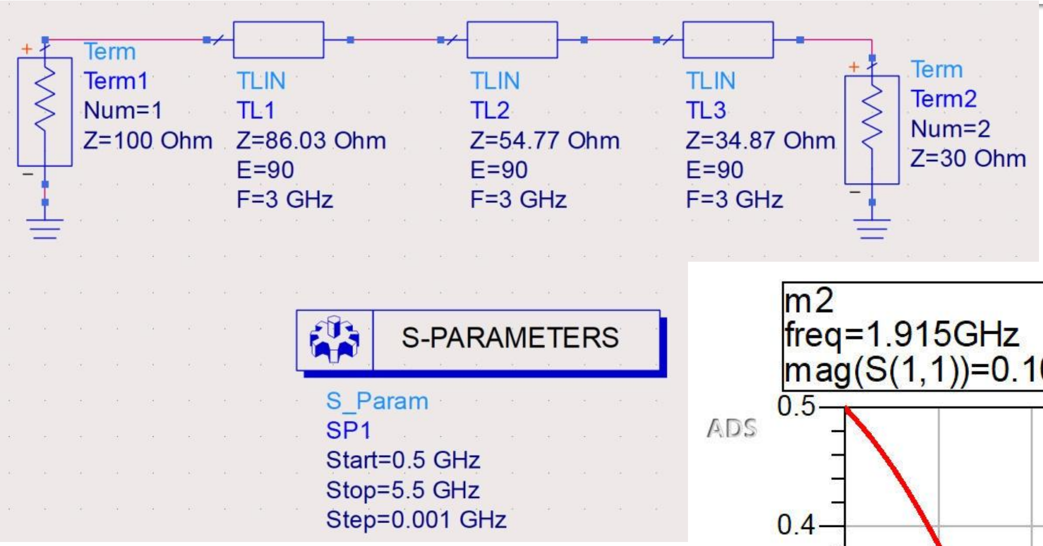
- $|\Gamma|^2$ is a power reflection coefficient

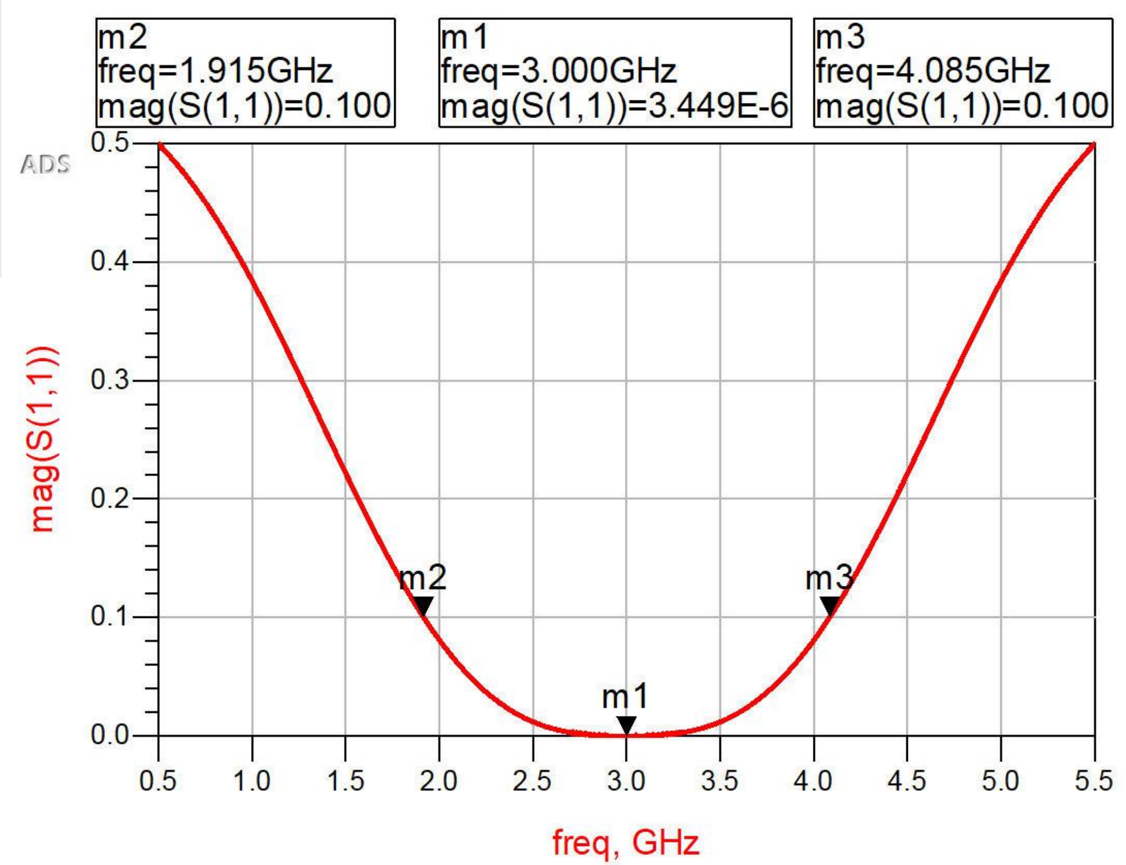
The quarter-wave transformer



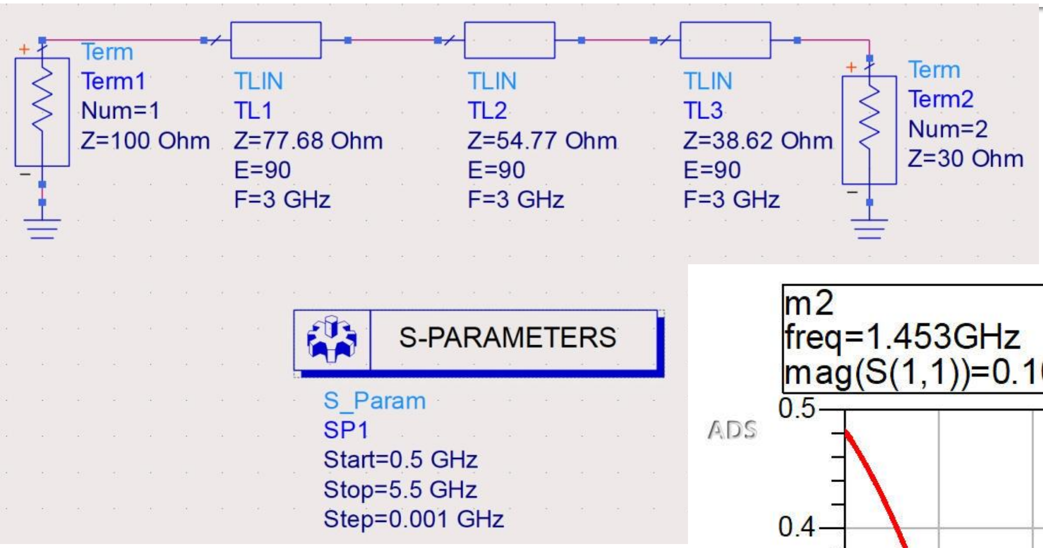


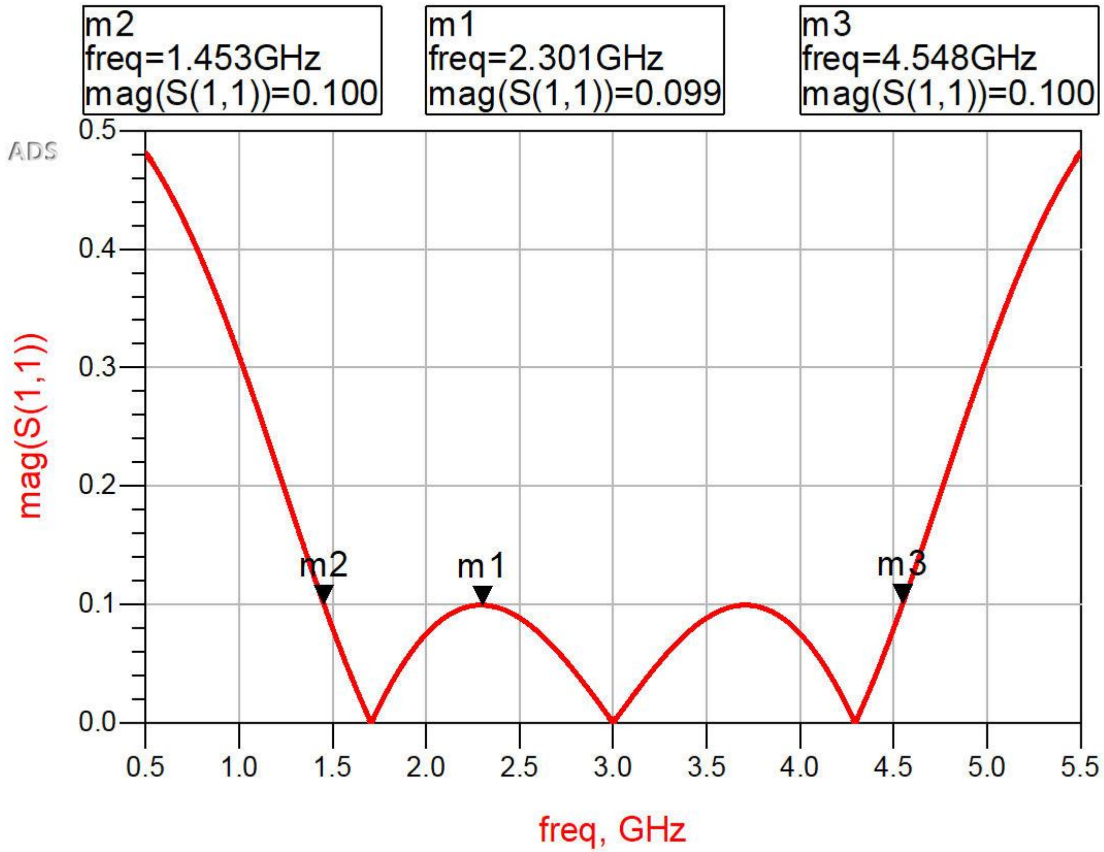
Binomial multisection transformer





Chebyshev multisection transformer

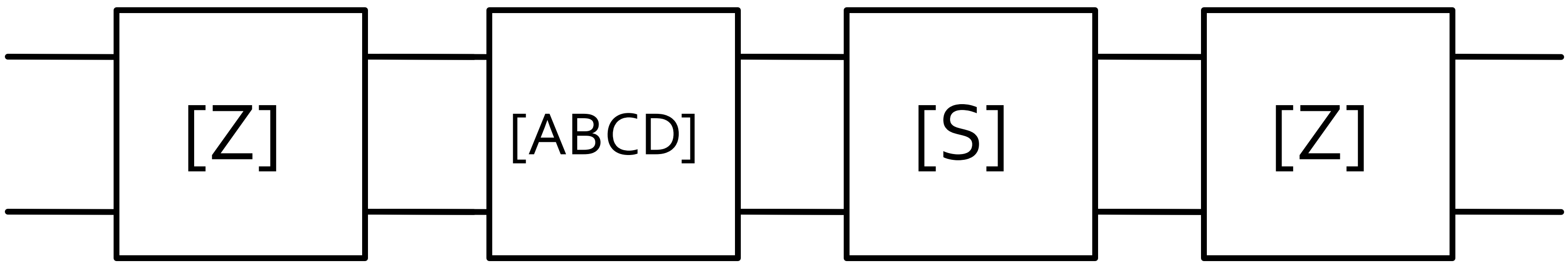




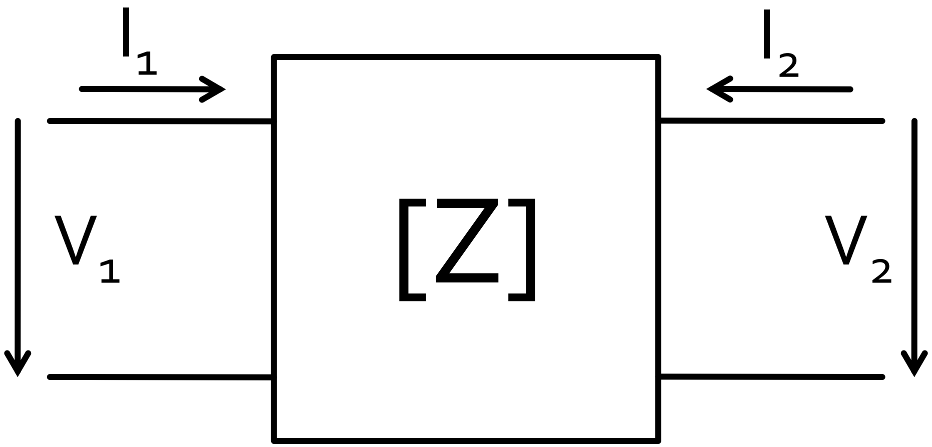
Microwave Network Analysis

Network Analysis

- We try to separate a complex circuit into individual blocks
- These are analyzed separately (decoupled from the rest of the circuit) and are characterized only by the port level signals (**black box**)
- Network-level analysis allows you to put together individual block results and get a total result for the entire circuit



Impedance matrix – Z



$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} \cdot \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

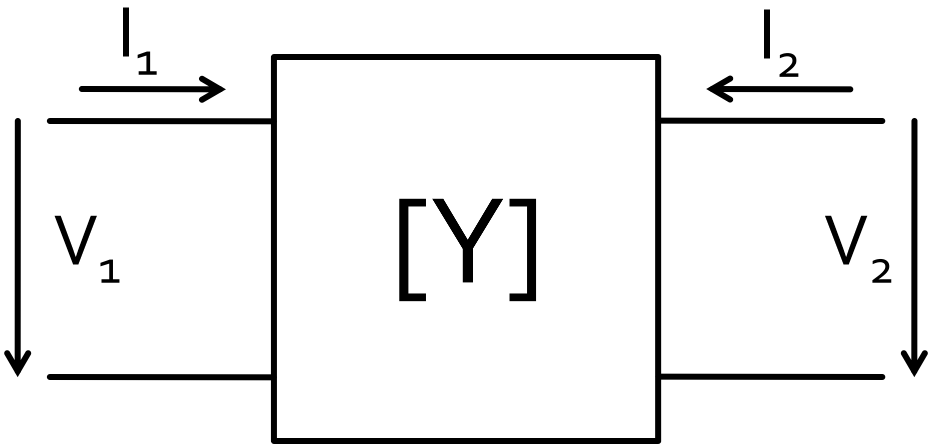
$$V_1 = Z_{11} \cdot I_1 + Z_{12} \cdot I_2$$
$$V_2 = Z_{21} \cdot I_1 + Z_{22} \cdot I_2$$

$$V_1 = Z_{11} \cdot I_1 \Big|_{I_2=0} \qquad Z_{11} = \frac{V_1}{I_1}\Big|_{I_2=0}$$

- Z11 – input impedance with open-circuited output

$$Z_{11} = \frac{V_1}{I_1}\Big|_{I_2=0} \qquad Z_{12} = \frac{V_1}{I_2}\Big|_{I_1=0} \qquad Z_{21} = \frac{V_2}{I_1}\Big|_{I_2=0} \qquad Z_{22} = \frac{V_2}{I_2}\Big|_{I_1=0}$$

Admittance matrix – Y



$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} \cdot \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

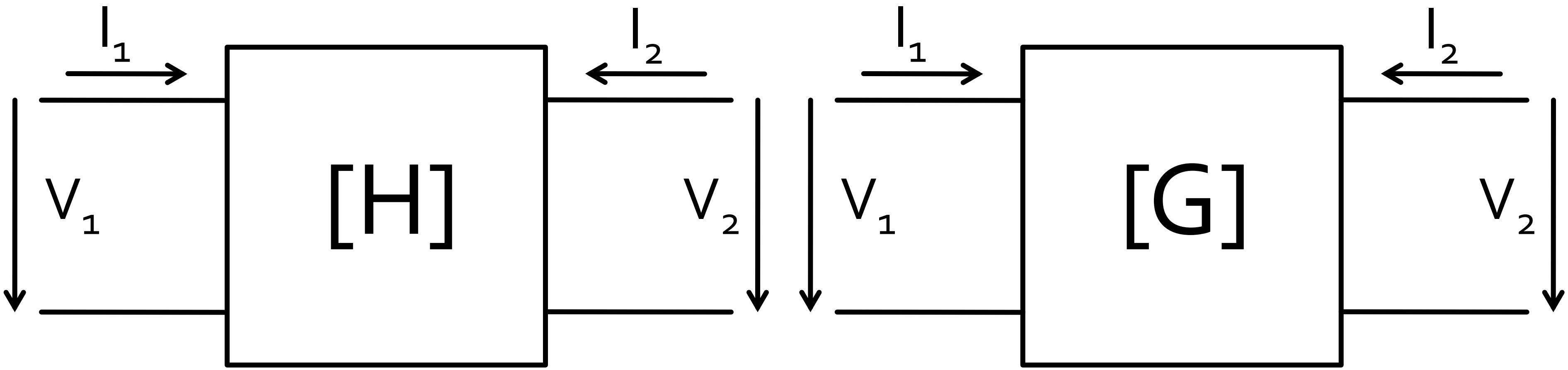
$$I_1 = Y_{11} \cdot V_1 + Y_{12} \cdot V_2$$
$$I_2 = Y_{21} \cdot V_1 + Y_{22} \cdot V_2$$

$$I_1 = Y_{11} \cdot V_1 \Big|_{V_2=0} \qquad Y_{11} = \frac{I_1}{V_1}\Big|_{V_2=0}$$

- Y11 – input admittance with short-circuited output

$$Y_{11} = \frac{I_1}{V_1}\Big|_{V_2=0} \qquad Y_{12} = \frac{I_1}{V_2}\Big|_{V_1=0} \qquad Y_{21} = \frac{I_2}{V_1}\Big|_{V_2=0} \qquad Y_{22} = \frac{I_2}{V_2}\Big|_{V_1=0}$$

Hybrid matrices – H and G



$$\begin{bmatrix} V_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} H_{11} & H_{12} \\ H_{21} & H_{22} \end{bmatrix} \cdot \begin{bmatrix} I_1 \\ V_2 \end{bmatrix} \qquad \begin{bmatrix} I_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} G_{11} & G_{12} \\ G_{21} & G_{22} \end{bmatrix} \cdot \begin{bmatrix} V_1 \\ I_2 \end{bmatrix}$$

$$H_{21} = \left.\frac{I_2}{I_1}\right|_{V_2=0\, sau\, H_{22}\to\infty}$$

- h_{21E} widely used for Bipolar Transistors, common emitter topology (or β, h_{22E} >>)

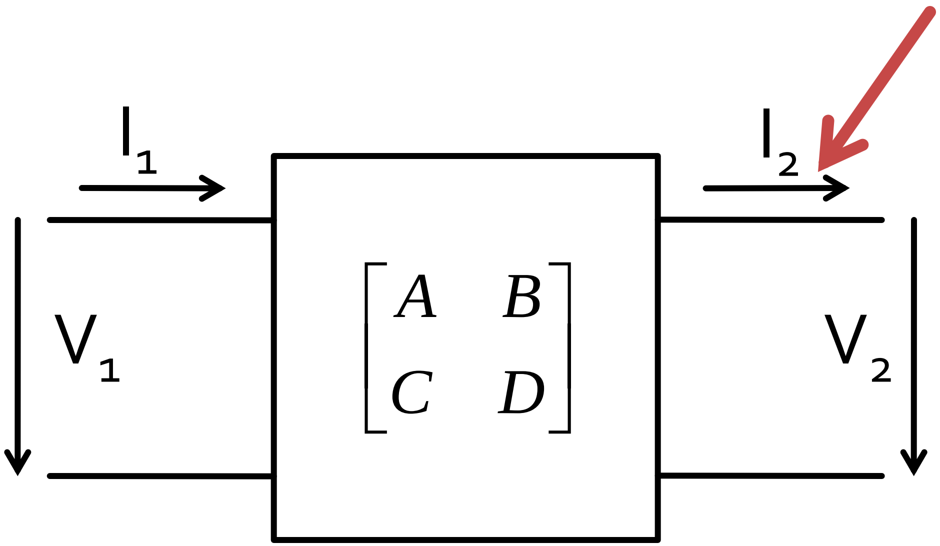
Network Analysis

- Each matrix is best suited for a particular mode of port excitation (V, I)
  - matrix H in common emitter connection for TB: I_B, V_{CE}
  - matrices provide the associated quantities depending on the “attack” ones
- Traditional notation of Z, Y, G, H parameters is in lowercase (z, y, g, h)
- In microwave analysis we prefer the notation in uppercase to avoid confusion with the normalized parameters

$$z = \frac{Z}{Z_0} \qquad y = \frac{Y}{Y_0} = \frac{1/Z}{1/Z_0} = \frac{Z_0}{Z} = Z_0 \cdot Y$$

$$z_{11} = \frac{Z_{11}}{Z_0} \qquad y_{11} = \frac{Y_{11}}{Y_0} = Z_0 \cdot Y_{11}$$

ABCD (transmission) matrix



$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \cdot \begin{bmatrix} V_2 \\ I_2 \end{bmatrix}$$

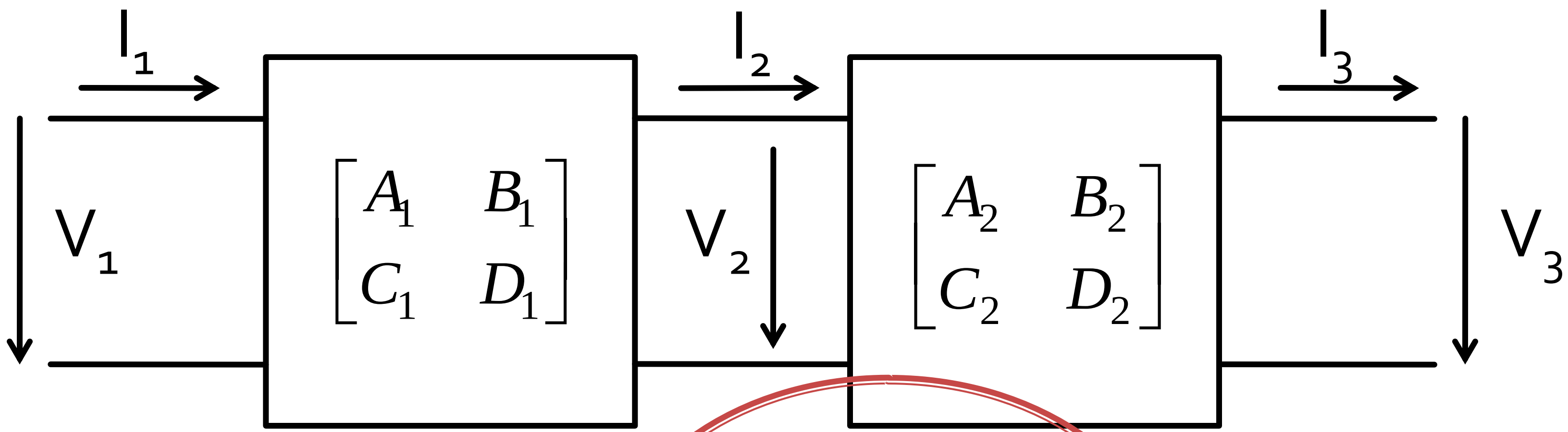
$$V_1 = A \cdot V_2 + B \cdot I_2$$
$$I_1 = C \cdot V_2 + D \cdot I_2$$

$$\begin{bmatrix} V_2 \\ I_2 \end{bmatrix} = \frac{1}{A \cdot D - B \cdot C} \cdot \begin{bmatrix} D & -B \\ -C & A \end{bmatrix} \cdot \begin{bmatrix} V_1 \\ I_1 \end{bmatrix}$$

$$A = \left. \frac{V_1}{V_2} \right|_{I_2 = 0} \qquad B = \left. \frac{V_1}{I_2} \right|_{V_2 = 0} \qquad C = \left. \frac{I_1}{V_2} \right|_{I_2 = 0} \qquad D = \left. \frac{I_1}{I_2} \right|_{V_2 = 0}$$

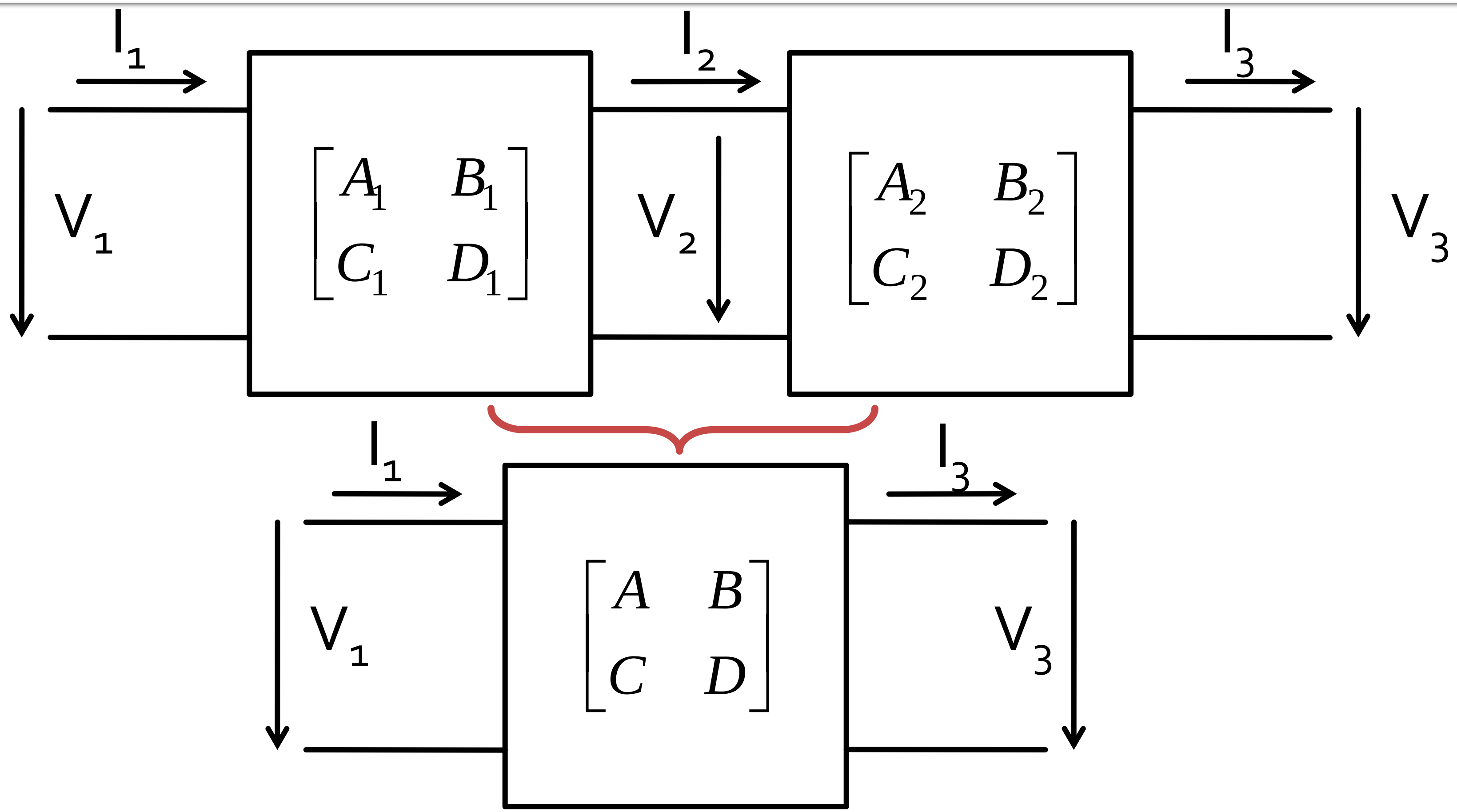
ABCD (transmission) matrix

- This 2X2 matrix characterizes the "input"/"output" relation
- Allows easy chaining of multiple two-ports



$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A_1 & B_1 \\ C_1 & D_1 \end{bmatrix} \cdot \begin{bmatrix} V_2 \\ I_2 \end{bmatrix} = \begin{bmatrix} A_1 & B_1 \\ C_1 & D_1 \end{bmatrix} \cdot \begin{bmatrix} A_2 & B_2 \\ C_2 & D_2 \end{bmatrix} \cdot \begin{bmatrix} V_3 \\ I_3 \end{bmatrix}$$

ABCD (transmission) matrix



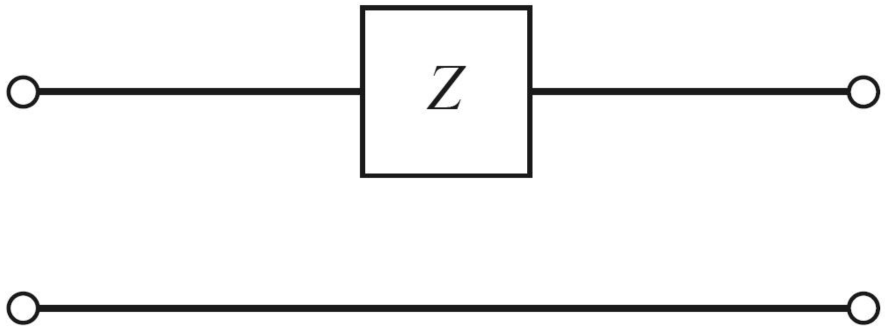
$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} A_1 & B_1 \\ C_1 & D_1 \end{bmatrix} \cdot \begin{bmatrix} A_2 & B_2 \\ C_2 & D_2 \end{bmatrix}$$

ABCD (transmission) matrix

- suitable **only** for two-port networks (Z, Y can be easily extended for multiport / n-ports)
- allows easy coupling of multiple elements
- allows the calculation of complex circuits with one input and one output by breaking them in individual component blocks
- a library of ABCD matrices for elementary two-port networks can be built up

Library of ABCD matrices

- Series impedance



$$A = 1 \qquad B = Z$$
$$C = 0 \qquad D = 1$$

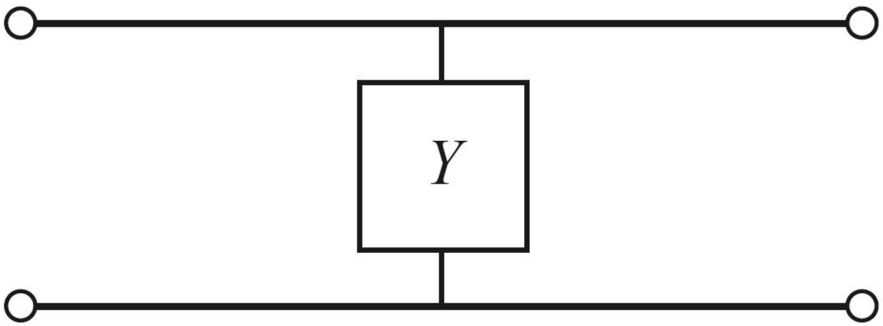
$$\begin{bmatrix} 1 & Z \\ 0 & 1 \end{bmatrix}$$

$$A = \left.\frac{V_1}{V_2}\right|_{I_2=0} = 1 \qquad B = \left.\frac{V_1}{I_2}\right|_{V_2=0} = \frac{V_1}{V_1/Z} = Z$$

$$C = \left.\frac{I_1}{V_2}\right|_{I_2=0} = 0 \qquad D = \left.\frac{I_1}{I_2}\right|_{V_2=0} = \frac{I_1}{I_1} = 1$$

Library of ABCD matrices

- Shunt admittance



$$A = 1 \qquad B = 0$$

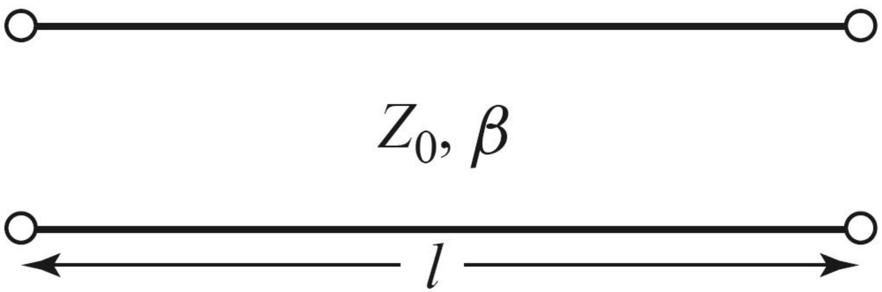
$$C = Y \qquad D = 1$$

$$\begin{bmatrix} 1 & 0 \\ Y & 1 \end{bmatrix}$$

Homework!

Library of ABCD matrices

- Transmission line



$$A = \cos \beta \cdot l$$

$$B = j \cdot Z_0 \cdot \sin \beta \cdot l$$

$$C = j \cdot Y_0 \cdot \sin \beta \cdot l$$

$$D = \cos \beta \cdot l$$

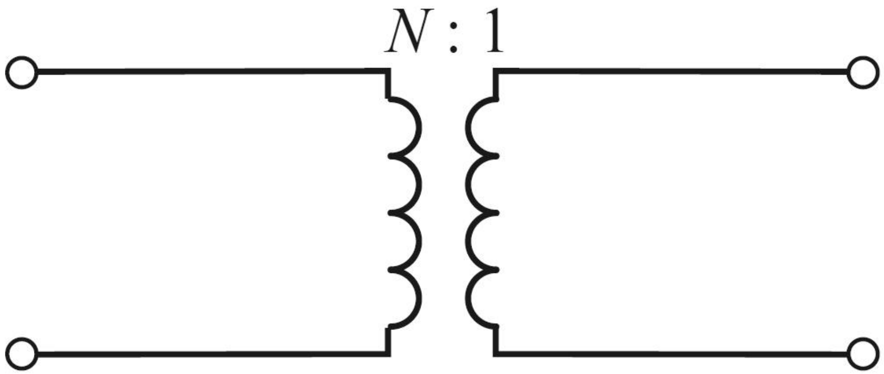
Homework!

$$\begin{bmatrix} \cos \beta \cdot l & j \cdot Z_0 \cdot \sin \beta \cdot l \\ j \cdot Y_0 \cdot \sin \beta \cdot l & \cos \beta \cdot l \end{bmatrix}$$

$$Z_{in} = Z_0 \cdot \frac{Z_L + j \cdot Z_0 \cdot \tan \beta \cdot l}{Z_0 + j \cdot Z_L \cdot \tan \beta \cdot l}$$

Library of ABCD matrices

- Transformer



$$A = N \qquad B = 0$$

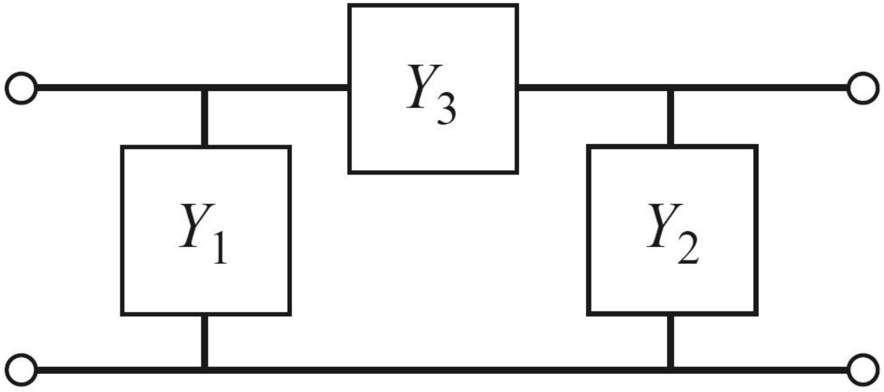
$$C = 0 \qquad D = \frac{1}{N}$$

$$\begin{bmatrix} N & 0 \\ 0 & \dfrac{1}{N} \end{bmatrix}$$

Homework!

Library of ABCD matrices

- π network



$$A = 1 + \frac{Y_2}{Y_3}$$

$$B = \frac{1}{Y_3}$$

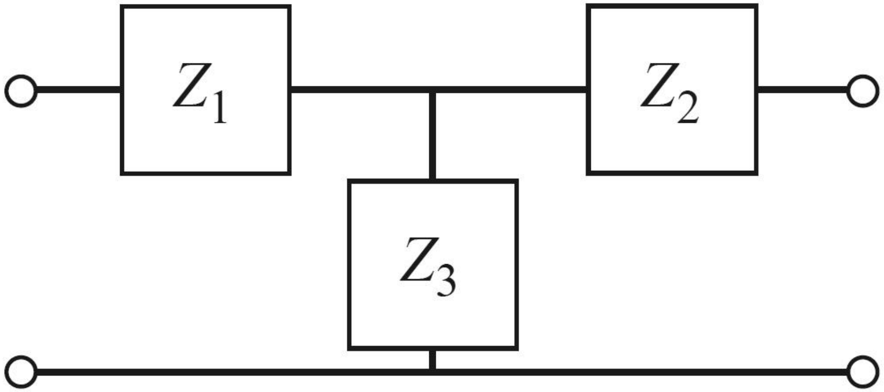
$$C = Y_1 + Y_2 + \frac{Y_1 \cdot Y_2}{Y_3}$$

$$D = 1 + \frac{Y_1}{Y_3}$$

Homework!

Library of ABCD matrices

- T network



$$A = 1 + \frac{Z_1}{Z_3}$$

$$B = Z_1 + Z_2 + \frac{Z_1 \cdot Z_2}{Z_3}$$

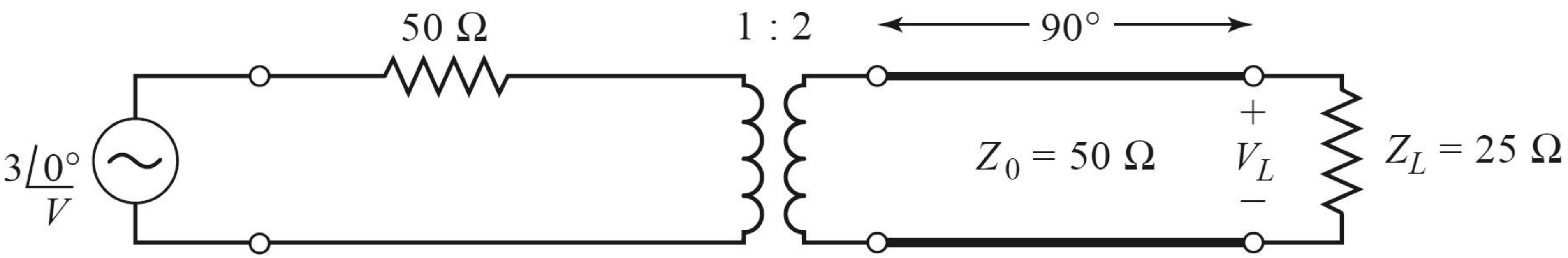
$$C = \frac{1}{Z_3}$$

$$D = 1 + \frac{Z_2}{Z_3}$$

Homework!

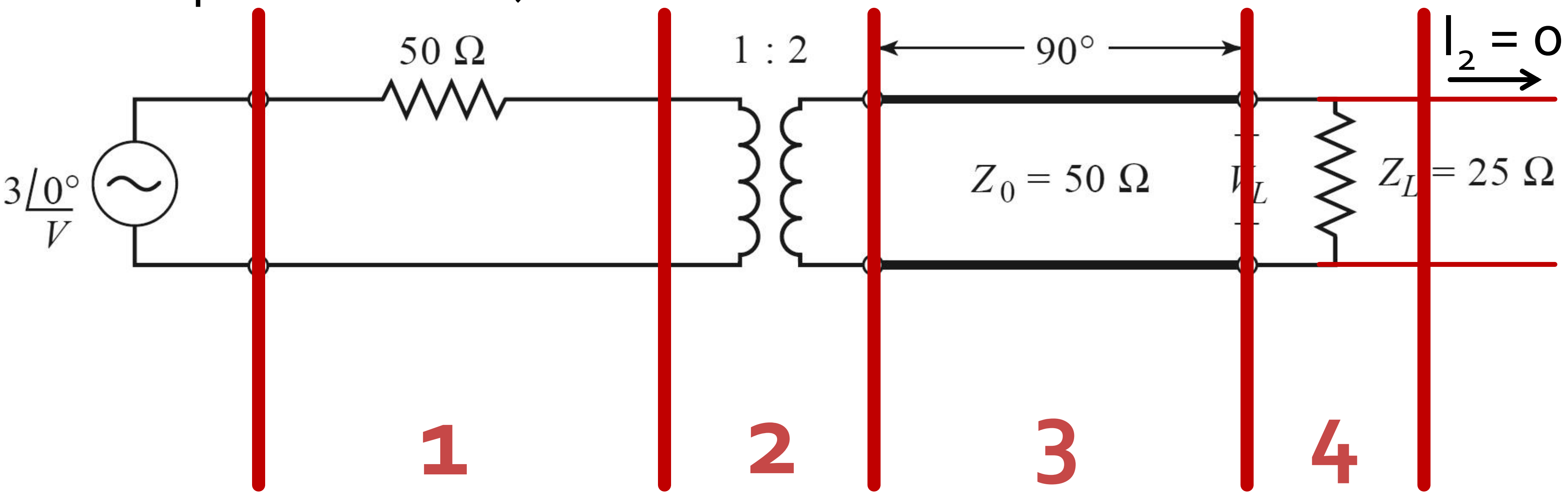
Example for ABCD matrix

- Find the voltage V_L across the load resistor in the circuit shown below



Example for ABCD matrix

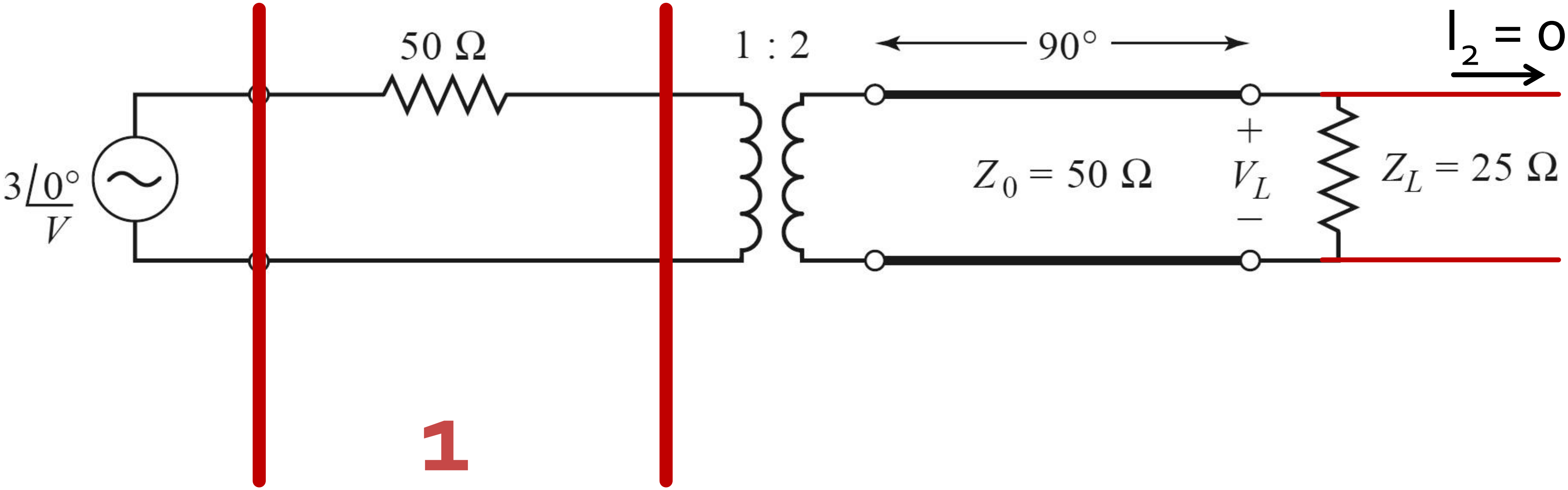
- We break the circuit in elementary sections
- Sources are left outside
- If necessary, input and output ports are created (and left open-circuited)



$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = M_1 \cdot M_2 \cdot M_3 \cdot M_4 \qquad V_1 = A \cdot V_2 + B \cdot I_2 \Big|_{I_2=0} \qquad V = A \cdot V_L \rightarrow V_L = \frac{V}{A}$$

Example for ABCD matrix

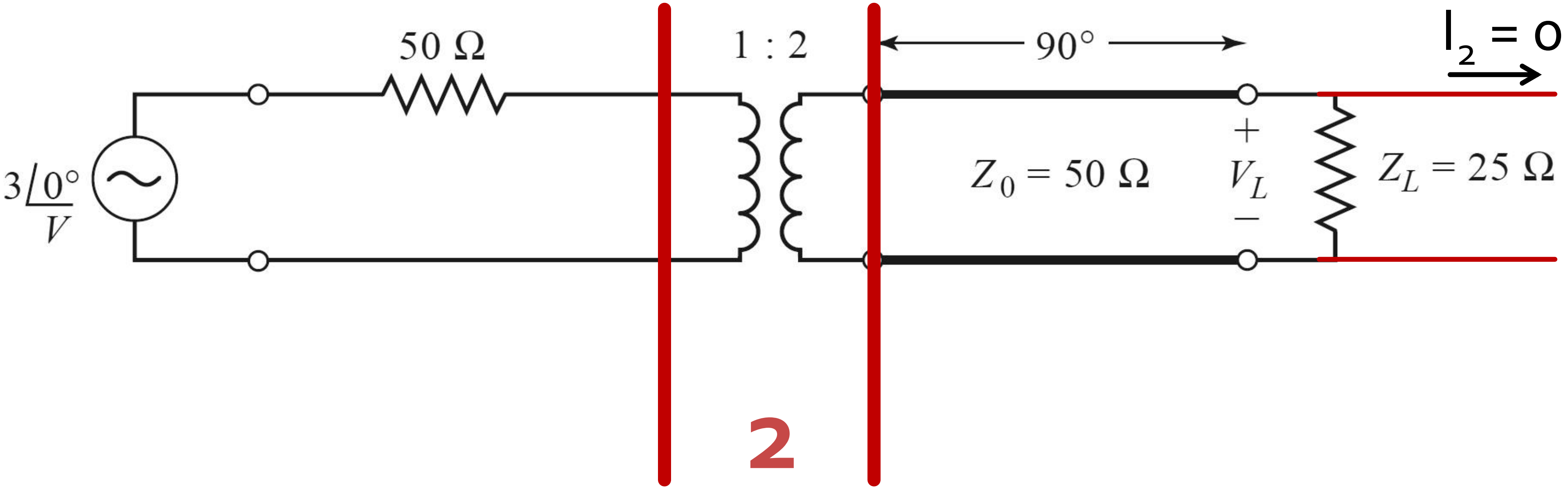
- M_1 , series impedance



$$M_1 = \begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 1 & 50 \\ 0 & 1 \end{bmatrix}$$

Example for ABCD matrix

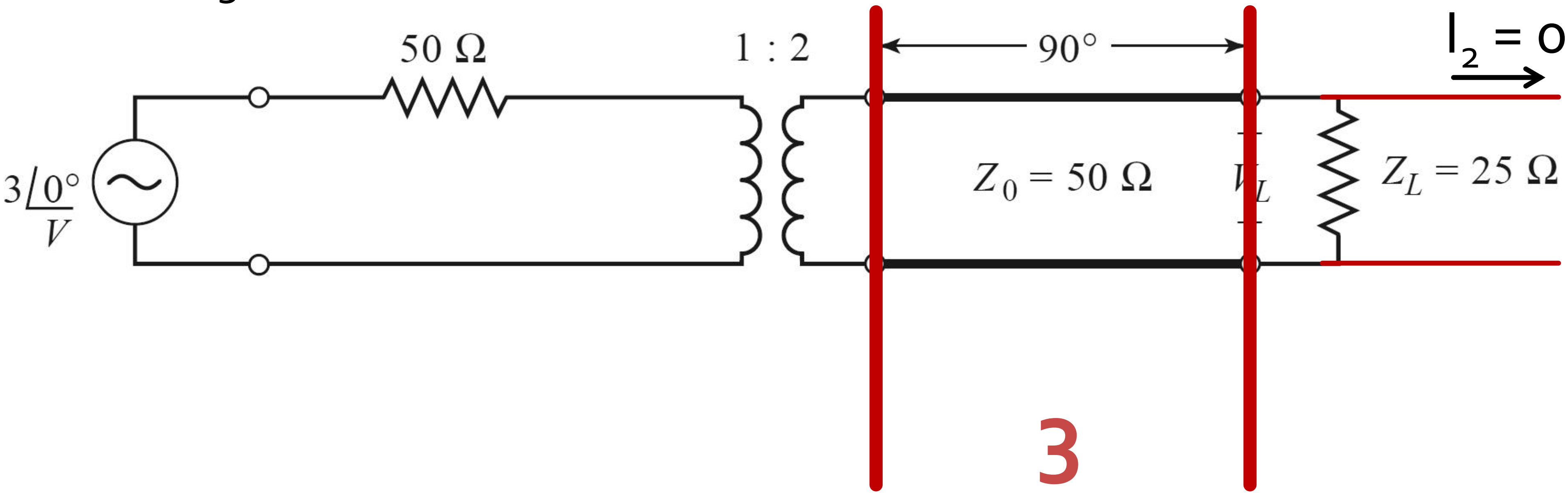
- M_2 , 1:2 transformer



$$M_2 = \begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 1/2 & 0 \\ 0 & 2 \end{bmatrix}$$

Example for ABCD matrix

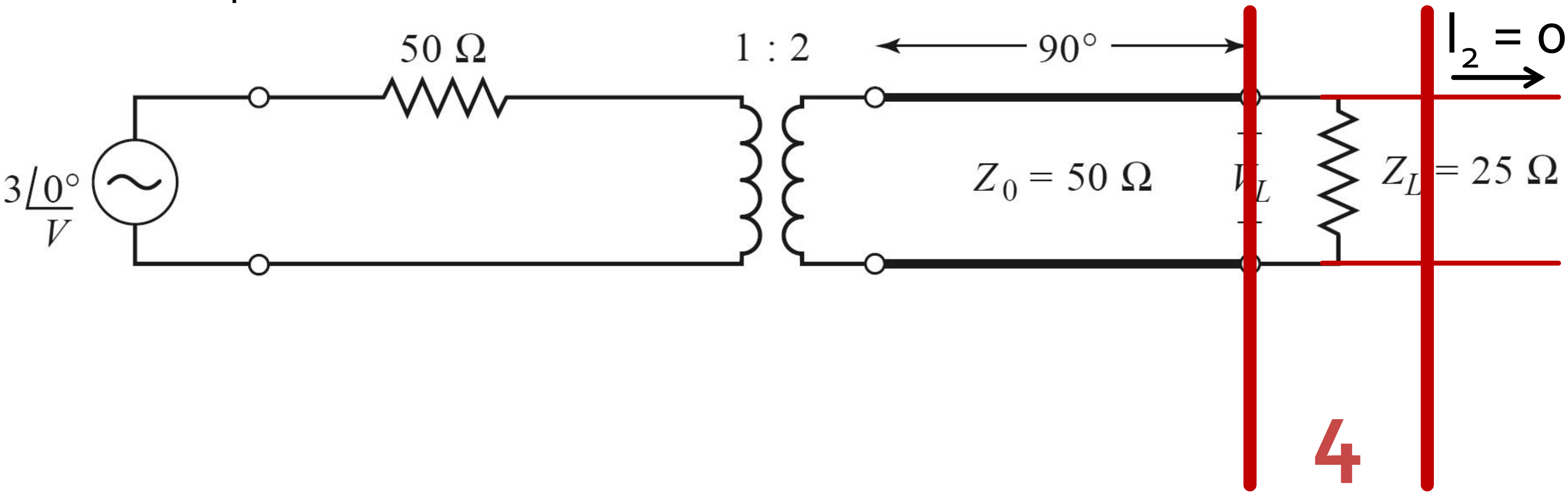
- M_3, series transmission line, E = 90°



$$M_3 = \begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 0 & 50 \cdot j \\ \dfrac{j}{50} & 0 \end{bmatrix}$$

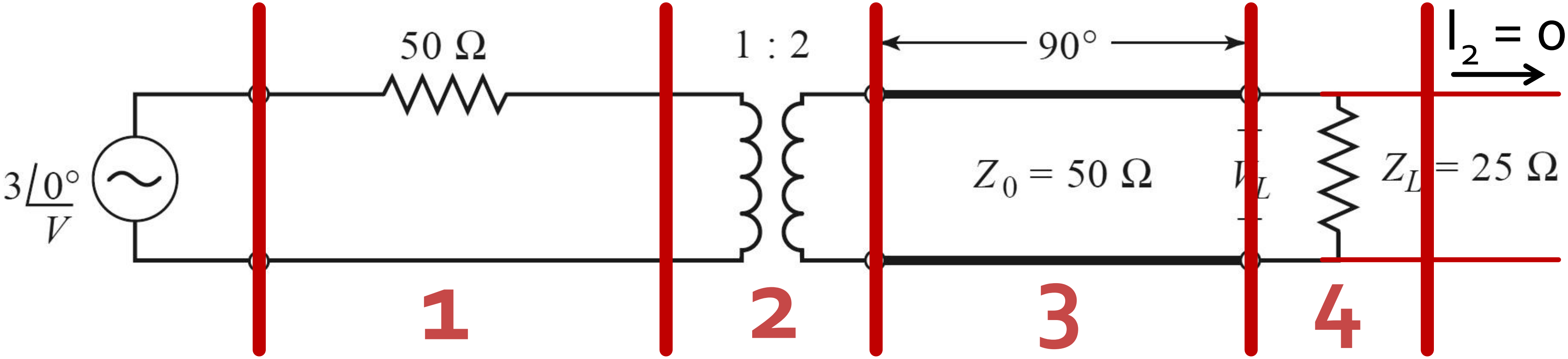
Example for ABCD matrix

- M_4 , shunt impedance/admittance



$$M_4 = \begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ \dfrac{1}{25} & 1 \end{bmatrix}$$

Example for ABCD matrix



$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 1 & 50 \\ 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} \frac{1}{2} & 0 \\ 0 & 2 \end{bmatrix} \cdot \begin{bmatrix} 0 & 50 \cdot j \\ \frac{j}{50} & 0 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 \\ \frac{1}{25} & 1 \end{bmatrix} = \begin{bmatrix} 3 \cdot j & 25 \cdot j \\ \frac{j}{25} & 0 \end{bmatrix}$$

$$V_L = \frac{V}{A} = \frac{3\angle 0°}{3 \cdot j} = 1\angle 90°$$

Contact

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